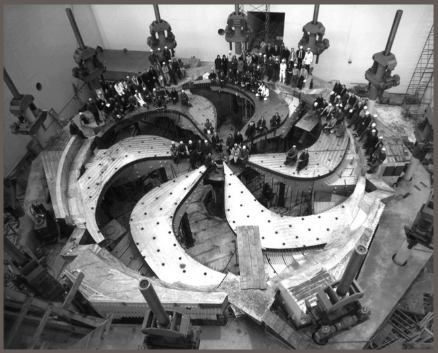
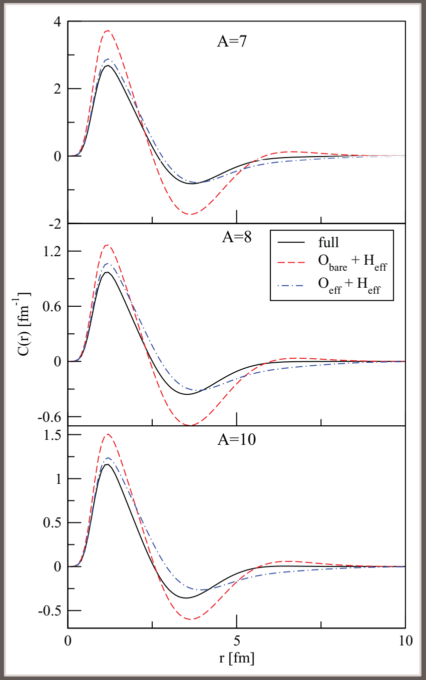
SRG and Valence-Space Renormalization of the 0νββ Decay Operator



TRIUMF Workshop on
"Interfacing theory and experiment for reliable double-beta decay matrix element calculations"
Vancouver, Canada, May 11-13, 2016.

Petr Navratil | TRIUMF



TRIUMF

Outline

- Motivation
  - *Ab initio* in nuclear physics
  - No-core shell model
    - GT transitions in ^{6}He ⟷ quenching
- SRG evolution of operators
- Okubo-Lee-Suzuki renormalization of operators in the valence space
  - Neutrinoless double beta decay toy model
- Outlook

TRIUMF

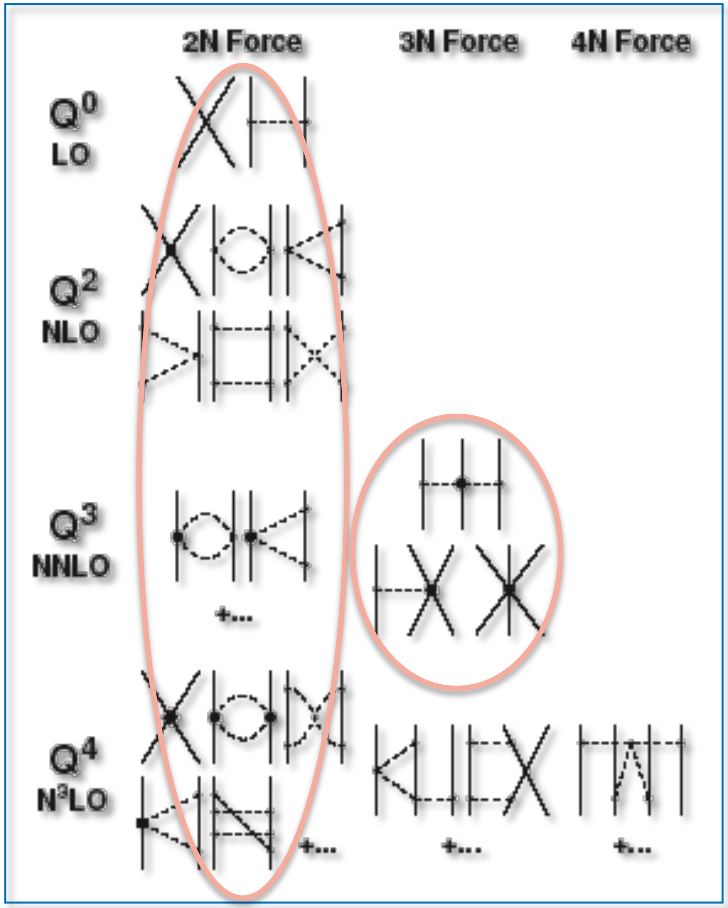
$M_{0\nu\beta\beta}$ (or any other) operator renormalization

- (i) Renormalization due to missing short-range correlations
  - Applies to many *ab initio* techniques
    - NCSM, CCM, IM-SRG…
  - Applies also to phenomenological approaches using effective interactions
  - SRG is the tool to do the renormalization (surely if SRG evolved interactions are used)
- (ii) Renormalization due to the valence space truncation
  - This is typically on top of the short-range renormalization (i)
  - *Ab initio*: Valence space IM-SRG, CCEI, NCSM with core, MBPT
  - Phenomenology (SM, IBM): effective charges, quenching, MBPT…

Ab initio calculations in nuclear physics

- ✧ All nucleons are active
- ✧ Exact Pauli principle
- ✧ Realistic inter-nucleon interactions
  - ✧ Accurate description of NN (and 3N) data
- ✧ Controllable approximations

INPUT:
Realistic inter-nucleon interactions
from **chiral perturbation theory**
(N^3LO) NN+ (N^2LO) 3N

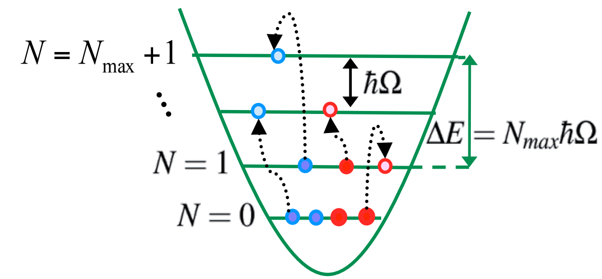


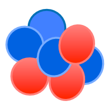
Softening of chiral NN+3N interactions
by **similarity renormalization group (SRG)**
unitary transformations:
Induce significant **3N interactions**
Induced 4N and higher much less important

$$H_\alpha = U_\alpha H U_\alpha^+ \Rightarrow \frac{dH_\alpha}{d\alpha} = [[T, H_\alpha], H_\alpha] \quad \left(\alpha = 1/\lambda^4\right)$$

No-core shell model

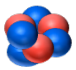
- No-core shell model (NCSM)
  - *A*-nucleon wave function expansion in the harmonic-oscillator (HO) basis
  - short- and medium range correlations
  - Bound-states, narrow resonances





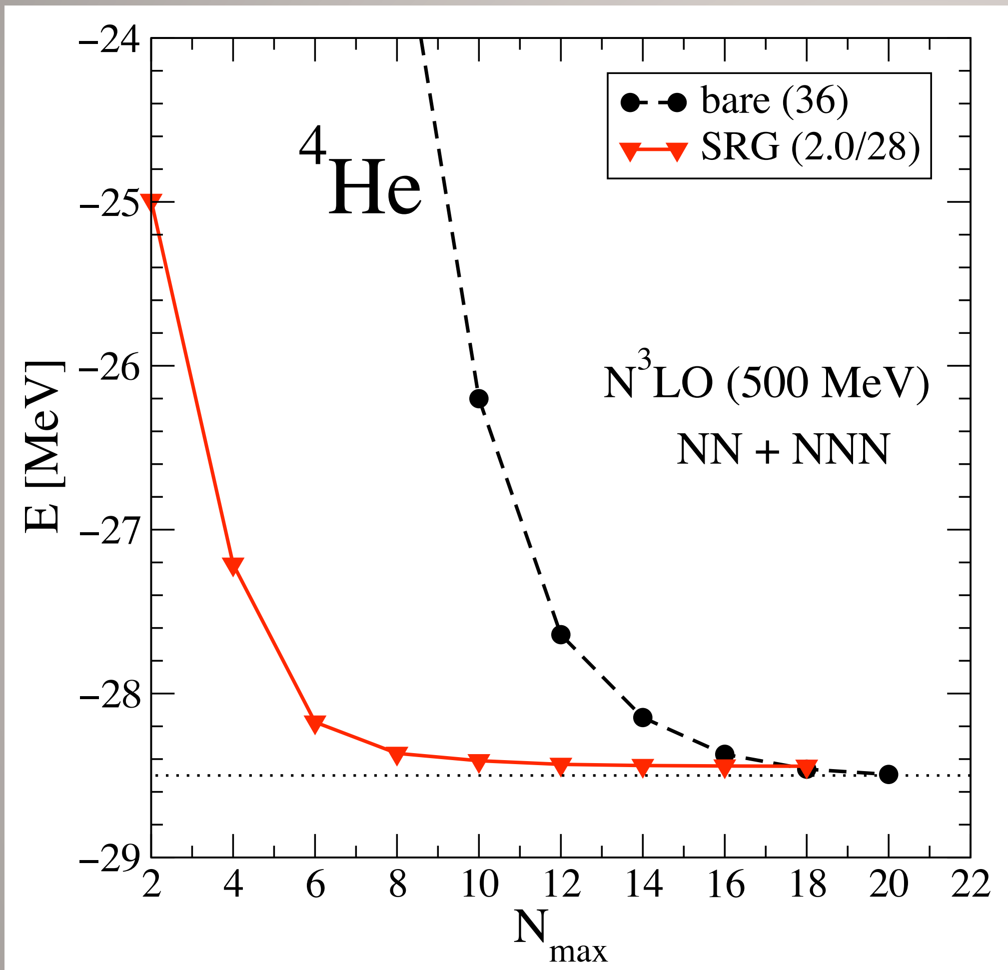
$$\Psi^A = \sum_{N=0}^{N_{\max}} \sum_i c_{Ni} \Phi^A_{Ni}$$

$$\Psi^{(A)} = \sum_\lambda c_\lambda \left|^{(A)} \ \cdot\ , \lambda \right\rangle$$



Unknowns

Calculations with chiral 3N: SRG renormalization needed



PRL 103, 082501 (2009) PHYSICAL REVIEW LETTERS week ending 21 AUGUST 2009

Evolution of Nuclear Many-Body Forces with the Similarity Renormalization Group

E. D. Jurgenson,[1] P. Navrátil,[2] and R. J. Furnstahl[1]

A=3 binding energy and half life constraint
c_D=-0.2, c_E=-0.205, Λ=500 MeV

Chiral N^{3}LO NN plus N^{2}LO NNN potential

- Bare interaction (black line)
  - Strong short-range correlations
    - Large basis needed
- SRG evolved effective interaction (red line)
  - Unitary transformation

$$H_\alpha = U_\alpha H U_\alpha^+ \Rightarrow \frac{dH_\alpha}{d\alpha} = [[T, H_\alpha], H_\alpha] \quad \left(\alpha = 1/\lambda^4\right)$$

  - Two- plus *three*-body components, *four*-body omitted
  - Softens the interaction
    - Smaller basis sufficient

Similarity Renormalization Group (SRG) evolution

- Continuous transformation driving Hamiltonian to band-diagonal form with respect to a chosen basis
- Unitary transformation $H_\alpha = U_\alpha H U_\alpha^+$ $\qquad U_\alpha U_\alpha^+ = U_\alpha^+ U_\alpha = 1$

$$\frac{dH_\alpha}{d\alpha} = \frac{dU_\alpha}{d\alpha} H U_\alpha^+ + U_\alpha H \frac{dU_\alpha^+}{d\alpha} = \frac{dU_\alpha}{d\alpha} U_\alpha^+ U_\alpha H U_\alpha^+ + U_\alpha H U_\alpha^+ U_\alpha \frac{dU_\alpha^+}{d\alpha}$$

$$= \frac{dU_\alpha}{d\alpha} U_\alpha^+ H_\alpha + H_\alpha U_\alpha \frac{dU_\alpha^+}{d\alpha} = \left[\eta_\alpha, H_\alpha\right]$$

$$\eta_\alpha \equiv \frac{dU_\alpha}{d\alpha} U_\alpha^+ = -\eta_\alpha^+$$

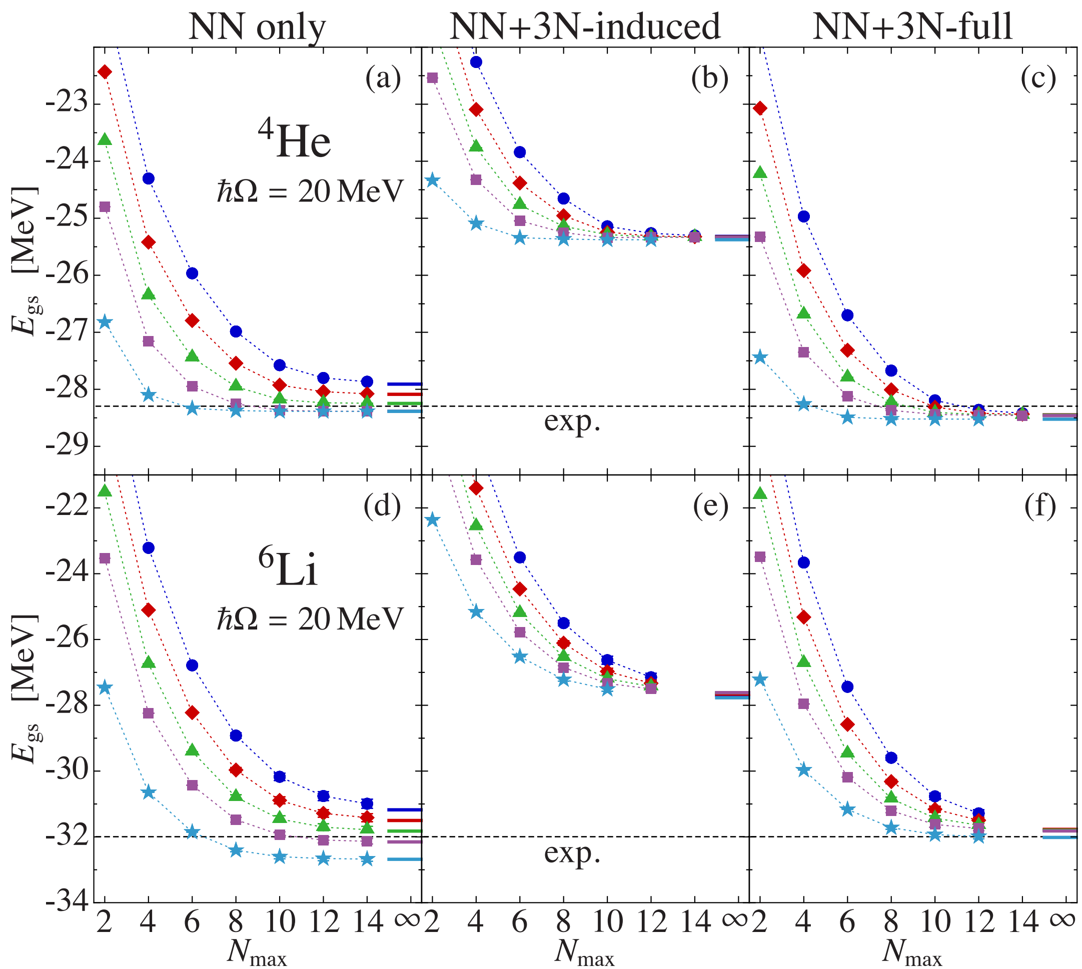
anti-Hermitian generator

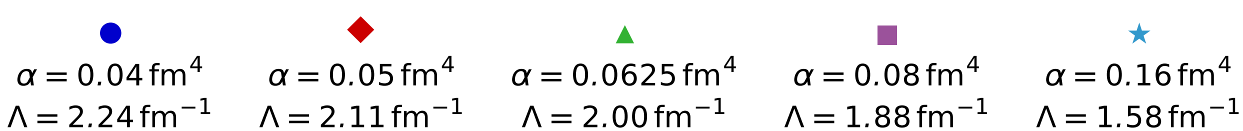
- Setting $\eta_\alpha = \left[G_\alpha, H_\alpha\right]$ with Hermitian G_α

$$\frac{dH_\alpha}{d\alpha} = \left[\left[G_\alpha, H_\alpha\right], H_\alpha\right]$$

- Customary choice in nuclear physics $G_\alpha = T$ …kinetic energy operator
  - band-diagonal in momentum space plane-wave basis
- Initial condition $H_{\alpha=0} = H_{\lambda=\infty} = H$ $\qquad \lambda^2 = 1/\sqrt{\alpha}$

Light nuclei with SRG evolved interactions





- Fast convergence
- Significant 3N induced interaction
- No 4N induced interaction

TRIUMF

^{6}He half-life

Precision measurement of ^{6}He beta decay

PHYSICAL REVIEW C **86**, 035506 (2012)

Precision measurement of the ^{6}He half-life and the weak axial current in nuclei

A. Knecht,[1,*] R. Hong,[1] D. W. Zumwalt,[1] B. G. Delbridge,[1] A. García,[1] P. Müller,[2] H. E. Swanson,[1] I. S. Towner,[3] S. Utsuno,[1] W. Williams,[2,†] and C. Wrede[1,‡]

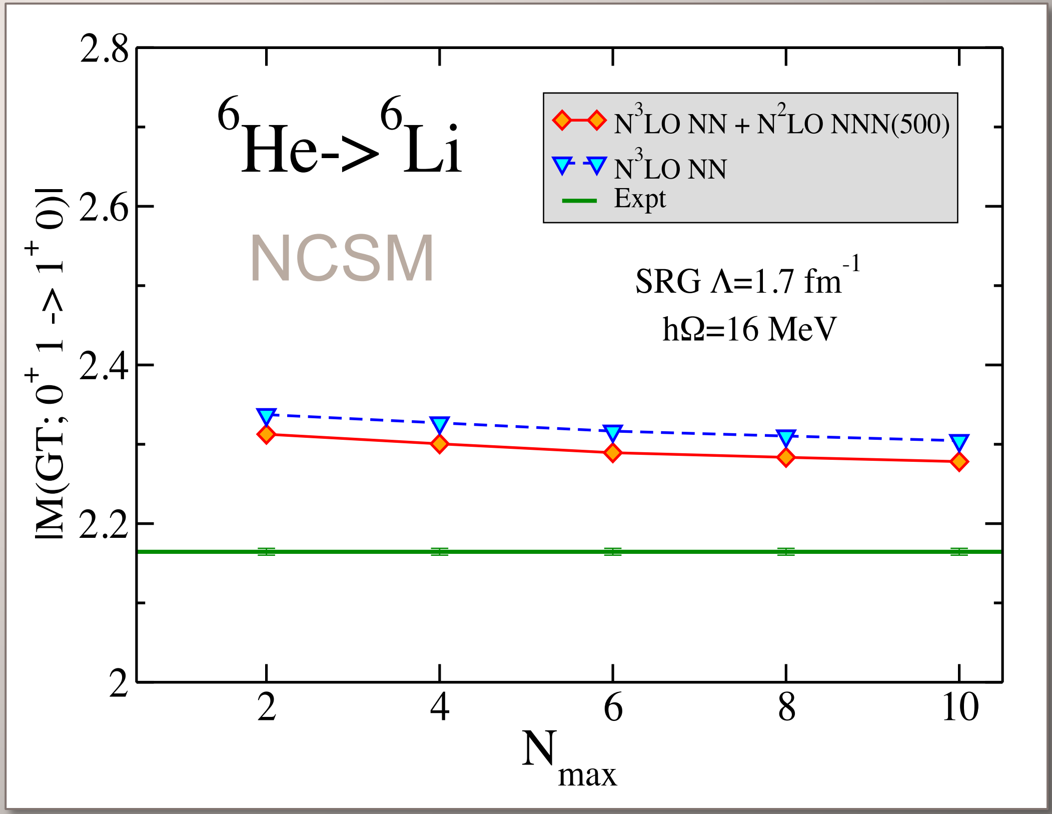
… challenge and test of *ab initio* calculations, nuclear forces and currents

Improvement with the NNN interaction

MEC must be included

Also:

Operator renormalization & continuum



TRIUMF

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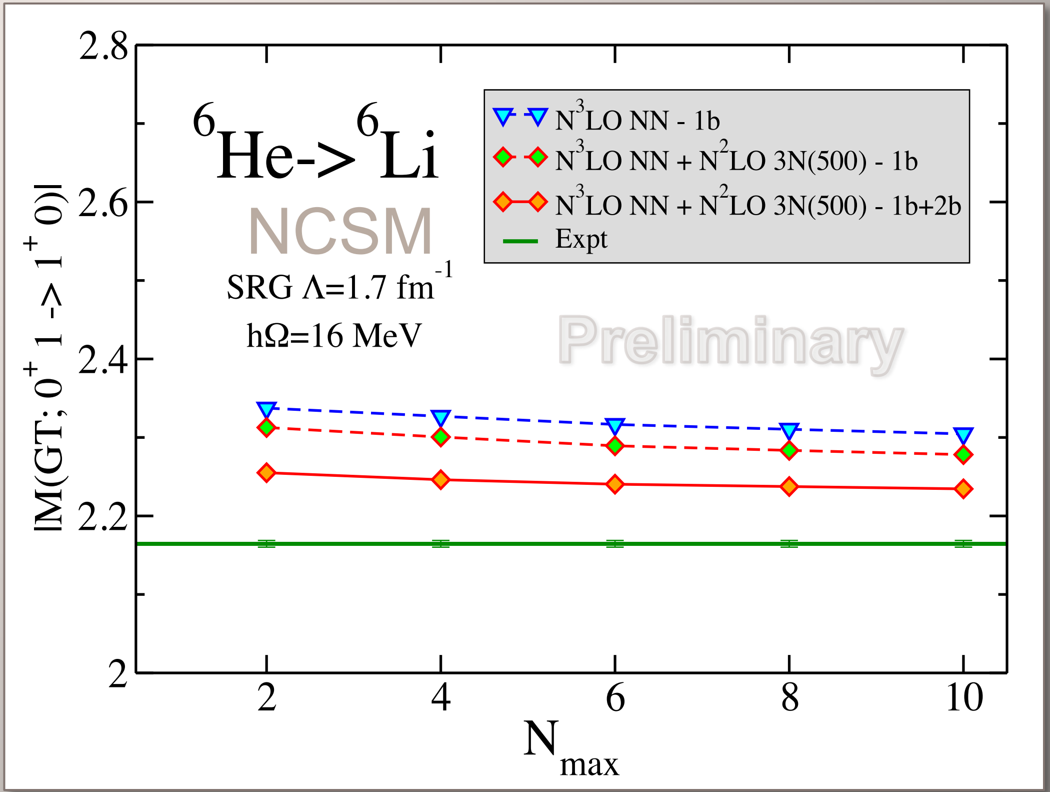

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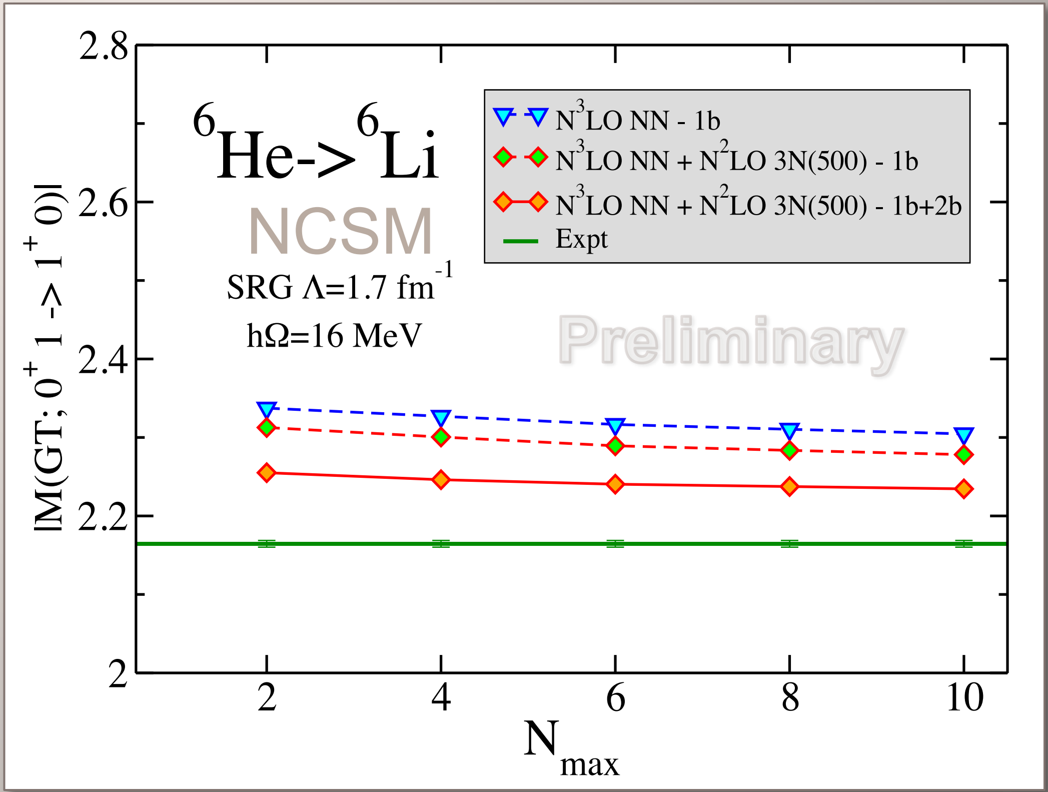


Improvement with the NNN interaction
Improvement with MEC
Still to be done:
Operator renormalization & continuum

SRG evolution of transition operators

PHYSICAL REVIEW C **90**, 011301(R) (2014)

Operator evolution for *ab initio* theory of light nuclei

Micah D. Schuster,[1,2] Sofia Quaglioni,[2] Calvin W. Johnson,[1] Eric D. Jurgenson,[2] and Petr Navrátil[3]

PHYSICAL REVIEW C **92**, 014320 (2015)

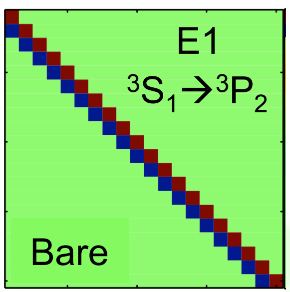
Operator evolution for *ab initio* electric dipole transitions of ^{4}He

Micah D. Schuster,[1,*] Sofia Quaglioni,[2,†] Calvin W. Johnson,[1,‡] Eric D. Jurgenson,[2] and Petr Navrátil[3]

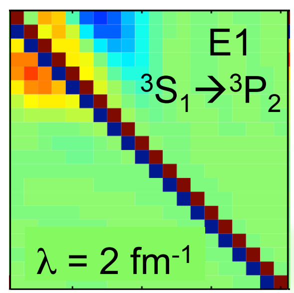
$$\hat{O}^{JT}_{\lambda} = \hat{U}^{f}_{\lambda}\hat{O}^{JT}_{\lambda=\infty}\hat{U}^{i*}_{\lambda}; \quad \hat{U}_{\lambda} = \sum_{\alpha} |\psi_{\alpha}(\lambda)\rangle\langle\psi_{\alpha}(\lambda=\infty)|$$

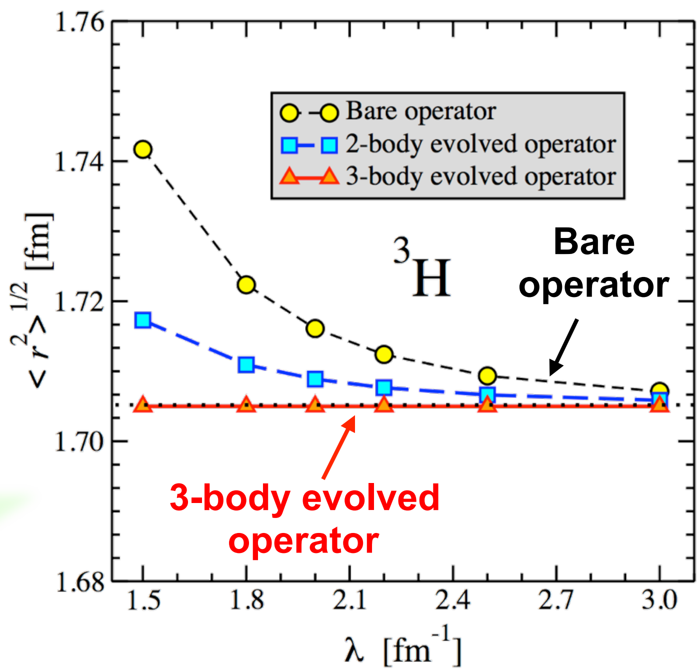
Final/initial unitary transformations

Eigenstates after & before evolution



Induces 2-body (& higher-body) operators





SRG evolution of transition operators

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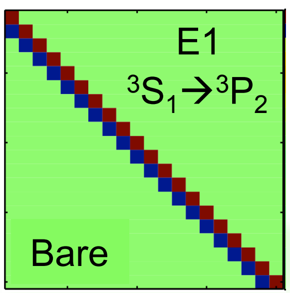

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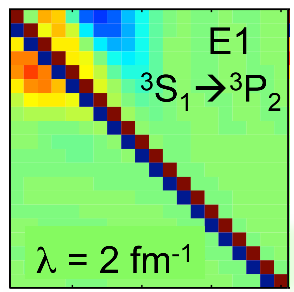

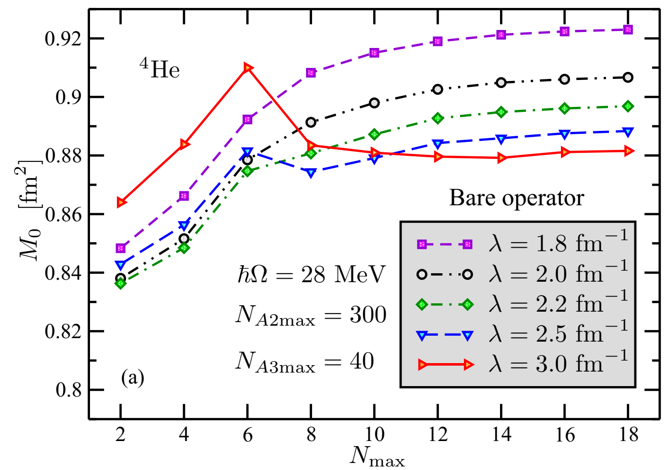

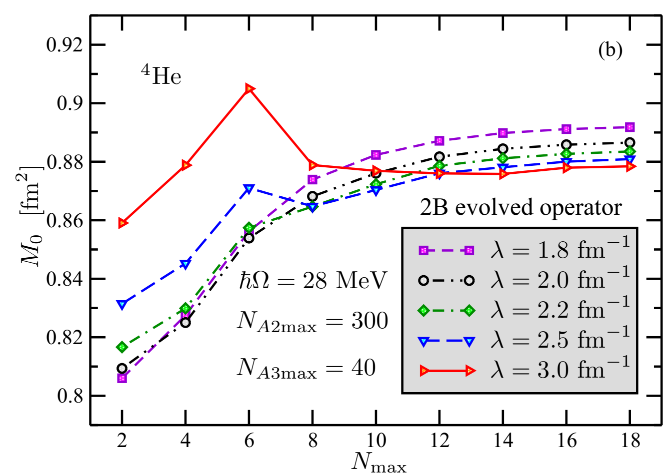

Total *E*1 dipole strength

$$M_0 = \langle \Psi_0 | \hat{D}^{+}\hat{D} | \Psi_0 \rangle = \left\| \hat{D} | \Psi_0 \rangle \right\|^2$$

2B evolved *E*1 operator $\hat{D}$

SRG evolution of transition operators

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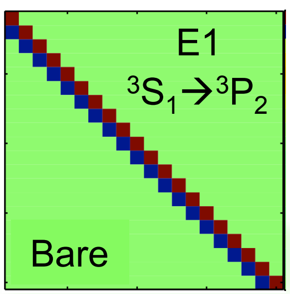

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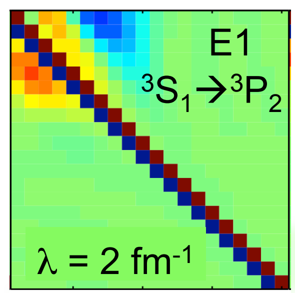

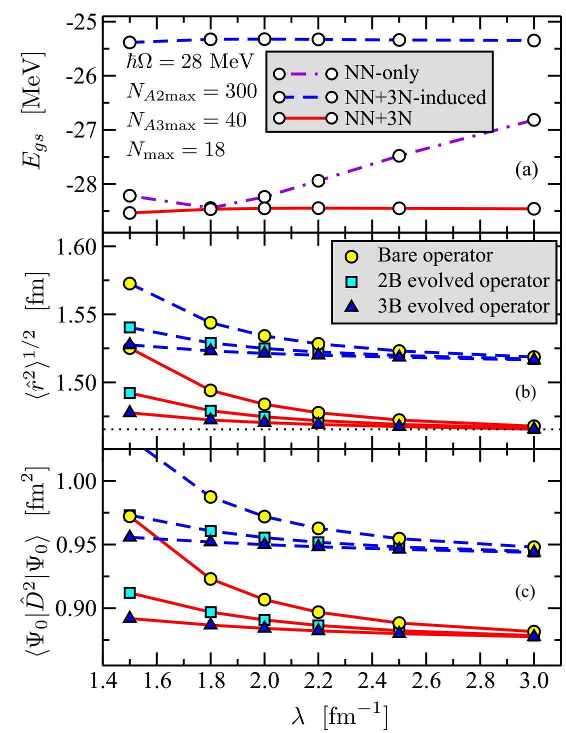

In ^{4}He, the inclusion of up to three-body induced terms all but completely restores the invariance of transitions under SRG

SRG evolution of transition operators

PHYSICAL REVIEW C **90**, 011301(R) (2014)

Operator evolution for *ab initio* theory of light nuclei

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PHYSICAL REVIEW C **92**, 014320 (2015)

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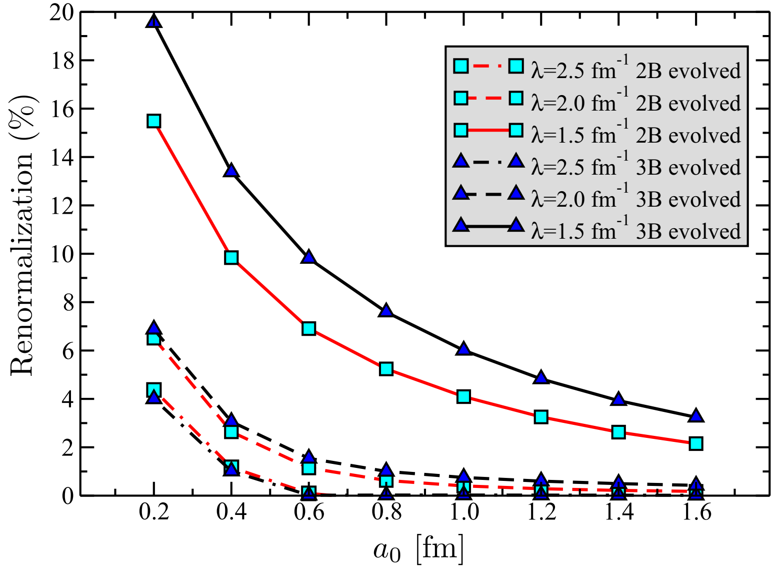
Final/initial unitary transformations

Eigenstates after & before evolution

$$\hat{O}(\vec{r}_1,\vec{r}_2) = \mathcal{A}\exp\left(-\frac{(\vec{r}_1-\vec{r}_2)^2}{a_0^2}\right)$$

$$\mathcal{A}\int \exp\left(-\frac{r^2}{a_0^2}\right)d\vec{r} = 1$$

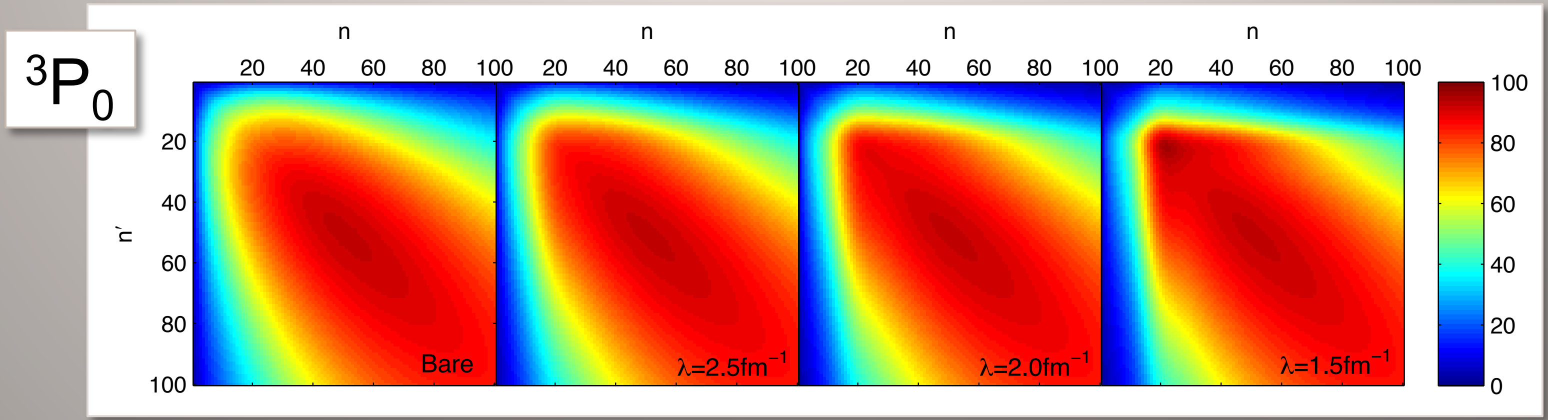


1) **The shorter the range the more renormalization**
2) **The 3B contribution *relatively* more important for the longer range**

Lesson for the neutrinoless double β decay: SRG evolve the $M_{0\nu\beta\beta}$ operator

SRG evolution of the $M_{0\nu\beta\beta}$ operator

- Applications in SM calculations presented by Mihai Horoi on Thursday
  - "0vββ Decay: To Quench or Not to Quench"
- 2B SRG evolution of the light neutrino 0vββ
  - chiral N^{3}LO NN, SRG λ=2 fm^{-1}
    - ~5% renormalization in ^{76}Ge
- 2B SRG evolution of the heavy neutrino 0vββ
  - chiral N^{3}LO NN, SRG λ=2 fm^{-1}
    - ~25% renormalization in ^{76}Ge



Operator renormalization in the valence space

PHYSICAL REVIEW C VOLUME 55, NUMBER 2 FEBRUARY 1997

Microscopic origins of effective charges in the shell model

Petr Navrátil,* Michael Thoresen, and Bruce R. Barrett

PHYSICAL REVIEW C **78**, 044302 (2008)

***Ab-initio* shell model with a core**

A. F. Lisetskiy,[1,*] B. R. Barrett,[1] M. K. G. Kruse,[1] P. Navratil,[2] I. Stetcu,[3] and J. P. Vary[4]

PHYSICAL REVIEW C **80**, 024315 (2009)

Effective operators from exact many-body renormalization

A. F. Lisetskiy,[1,2,*] M. K. G. Kruse,[1] B. R. Barrett,[1] P. Navratil,[3] I. Stetcu,[4] and J. P. Vary[5]

PHYSICAL REVIEW C **84**, 044316 (2011)

Nonperturbative renormalization of the neutrinoless double-β operator in p-shell nuclei

Deepshikha Shukla and Jonathan Engel
Department of Physics and Astronomy, University of North Carolina, Chapel Hill, North Carolina, 27516-3255, USA

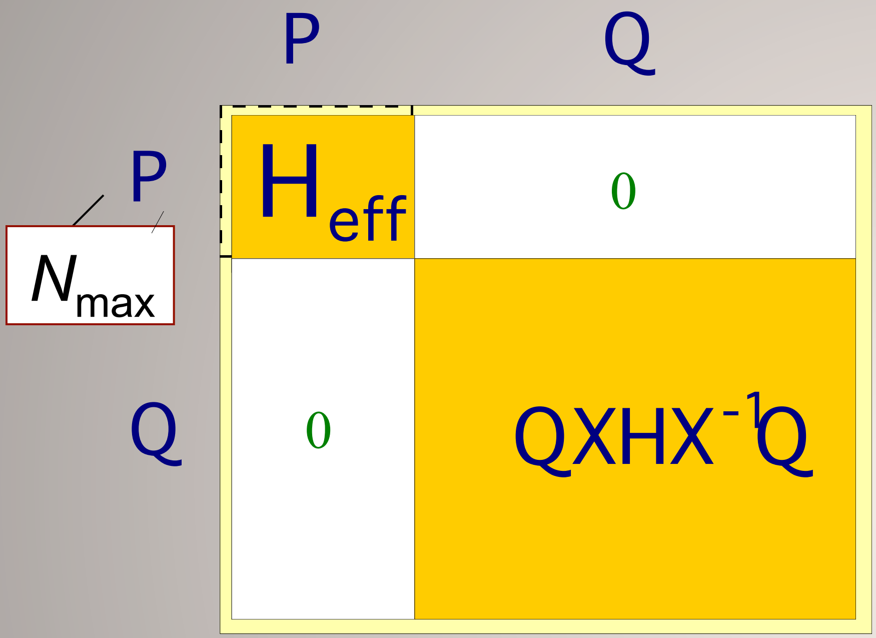
Petr Navratil
TRIUMF, 4004 Wesbrook Mall, Vancouver, British Columbia, V6T 2A3 Canada and
Lawrence Livermore National Laboratory, P.O. Box 808, L-414, Livermore, California 94551, USA

PRL **113,** 142502 (2014) PHYSICAL REVIEW LETTERS week ending 3 OCTOBER 2014

***Ab Initio* Coupled-Cluster Effective Interactions for the Shell Model: Application to Neutron-Rich Oxygen and Carbon Isotopes**

G. R. Jansen,[1,2] J. Engel,[3] G. Hagen,[1,2] P. Navratil,[4] and A. Signoracci[1,2]

Effective Hamiltonian & operators from Okubo-Lee-Suzuki transformation



$$H: \quad E_1, E_2, E_3, \ldots E_{d_P}, \ldots E_\infty$$

$$H_{\text{eff}}: \quad E_1, E_2, E_3, \ldots E_{d_P}$$

model space dimension

$$QXHX^{-1}P = 0$$

$$H_{\text{eff}} = PXHX^{-1}P$$

unitary $X=\exp[-\text{arctanh}(\omega^{+}-\omega)]$

$$\langle q|\,\omega\,|p\rangle = \sum_{k=0}^{d-1} \langle q|k\rangle\,\langle \underline{k}|p\rangle$$

$$M = P + \omega^\dagger\omega = P(1+\omega^\dagger\omega)P.$$

$$O_{\text{eff}} = M^{-\frac{1}{2}}(P+\omega^\dagger)O(P+\omega)M^{-\frac{1}{2}}$$

TRIUMF

Valence-space renormalization of the $M_{0\nu\beta\beta}$ operator

Toy problem: ^{6}He → ^{6}Be transition

NCSM calculations in N_{max}=6-10 space projected to N_{max}=0

Effective operator used for transitions in A=7,8,10 systems

$$\mathcal{M}_{fi} \equiv \langle f | \sum_{ab} M_{ab}^{GT} + M_{ab}^{F} + M_{ab}^{T} |i\rangle$$

$$M_{ab}^{GT} = H_{GT}(r_{ab}) \boldsymbol{\sigma}_a \cdot \boldsymbol{\sigma}_b ,$$

$$M_{ab}^{F} = H_F(r_{ab}) ,$$

$$H_K(r) = \frac{2R}{\pi r} \int_0^\infty \frac{h_K(q) \sin qr}{q + \bar{\omega}} dq , \quad K = GT, F.$$

$$\int_0^\infty C(r)\, dr = \mathcal{M}_{fi}$$

Valence-space renormalization of the $M_{0\nu\beta\beta}$ operator

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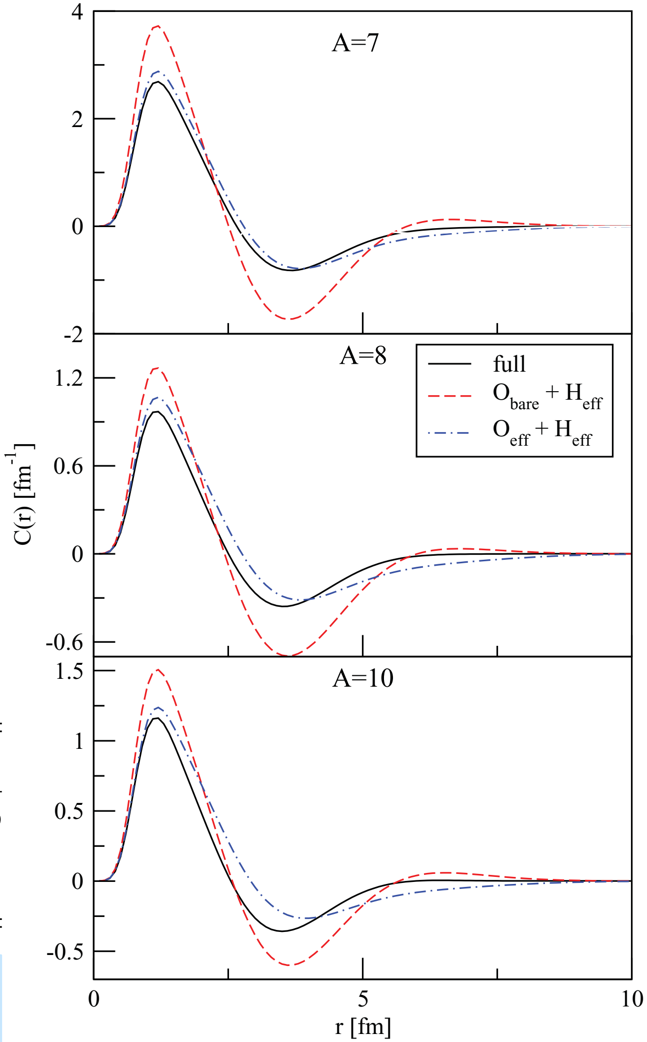
Effective operator used for transitions in A=7,8,10 systems:
$^{7,8,10}He \rightarrow {}^{7,8,10}Be$

$$\int_0^\infty C(r)\,dr = \mathcal{M}_{fi}$$

	7	8	10
full	1.76	0.48	0.79
bare	1.49	0.18	0.91
effective	1.90	0.59	1.23



Non-perturbative renormalization of the transition operator improves the shell model ability to reproduce *ab initio* results

TRIUMF

Conclusions and Outlook

- *Ab initio* calculations of nuclear structure and reactions is a dynamic field with significant advances

- Possible contribution to the neutrinoless double beta decay:
  - Renormalization of the transition operator to account for the short-range correlations using the SRG evolution
  - Renormalization to account for the valence-space truncation using the Okubo-Lee-Suzuki transformation and/or valence space IM-SRG
  - Benchmark calculations in ^{48}Ca and beyond

Collaborators contributing to presented results

Micah Shuster (ORNL)

Sofia Quaglioni, Eric Jurgenson (LLNL)

Calvin Johnson (SDSU)

Mihai Horoi (CMU)

Michael Desrochers (UBC), Doron Gazit (Hebrew U)

Jon Engel, D. Shukla (UNC)

A. Calci (TRIUMF), R. Roth (TU Darmstadt), D. Furnstahl (OSU)