Double beta-decay workshop
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Nucleon pairs in double-beta decay

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Outline

1. Neutrinoless double beta decay ^{48}Ca -> ^{48}Ti
2. Decay from neutron closed-shell nuclei
3. Decay from spherical open-shell nuclei
4. Open question on rotational nuclei

A large-scale shell model calculation on ^{48}Ca -> ^{48}Ti neutorinoless double beta decay

Y. Iwata et al., PRL 116, 112502 (2016)

Among many earlier studies, we carried out somewhat larger shell-model calculations in the sd+pf shell.

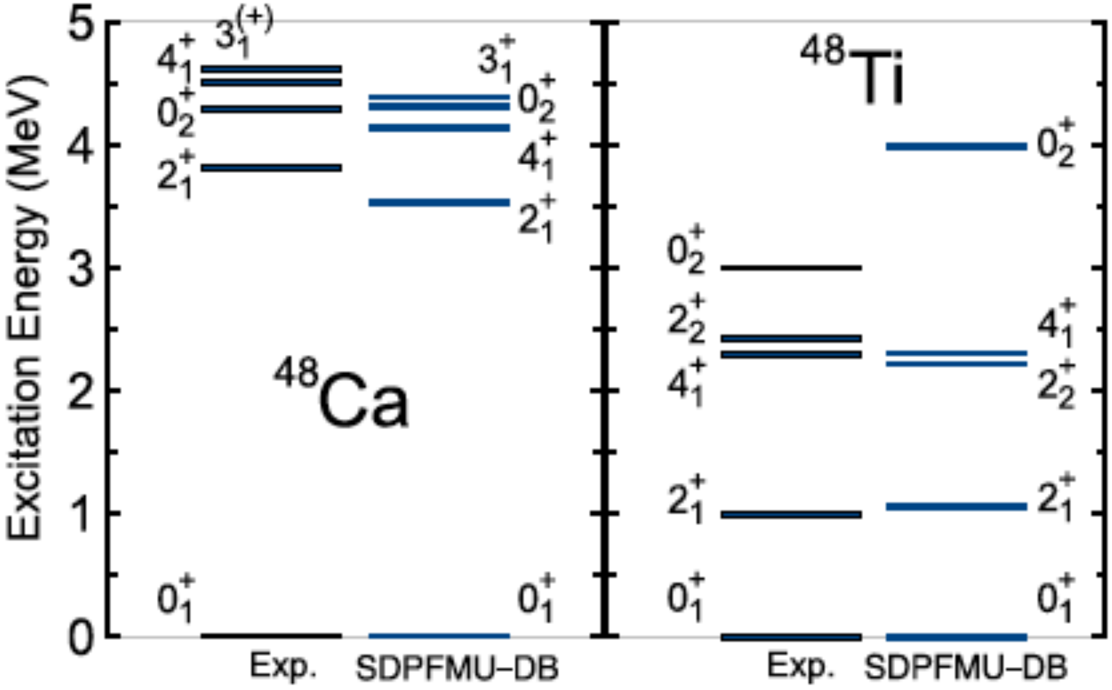


FIG. 1. Excitation spectra of ^{48}Ca and ^{48}Ti. The lowest five positive-parity states [41] are compared to *sdpf* calculations with the SDPFMU-DB interaction.

$$[T^{0\nu}_{1/2}(0^+_i \to 0^+_f)]^{-1} = G^{0\nu}|M^{0\nu}|^2\left(\frac{\langle m_{\beta\beta}\rangle}{m_e}\right)^2$$

$$M^{0\nu} = \langle 0^+_f|\hat{O}^{0\nu}|0^+_i\rangle = M^{0\nu}_{\mathrm{GT}} - \frac{g_{\mathrm{V}}^2}{g_{\mathrm{A}}^2}M^{0\nu}_{\mathrm{F}} + M^{0\nu}_{\mathrm{T}}$$

TABLE I. NME value for the ^{48}Ca $0\nu\beta\beta$ decay. The pf-shell calculation with GXPF1B is compared to the $sdpf$ $2\hbar\omega$ results obtained with the SDPFMU-DB and SDPFMU interactions. Total values ($M^{0\nu}$) are shown together with Gamow-Teller ($M^{0\nu}_{\mathrm{GT}}$), Fermi ($M^{0\nu}_{\mathrm{F}}$), and tensor ($M^{0\nu}_{\mathrm{T}}$) parts. Argonne- and CD-Bonn-type short-range correlations (SRC) are considered.

	GXPF1B				SDPFMU-DB				SDPFMU			
SRC	$M^{0\nu}_{\mathrm{GT}}$	$M^{0\nu}_{\mathrm{F}}$	$M^{0\nu}_{\mathrm{T}}$	$M^{0\nu}$	$M^{0\nu}_{\mathrm{GT}}$	$M^{0\nu}_{\mathrm{F}}$	$M^{0\nu}_{\mathrm{T}}$	$M^{0\nu}$	$M^{0\nu}_{\mathrm{GT}}$	$M^{0\nu}_{\mathrm{F}}$	$M^{0\nu}_{\mathrm{T}}$	$M^{0\nu}$
None	0.776	−0.216	−0.077	0.833	0.997	−0.304	−0.067	1.118	0.894	−0.291	−0.068	1.007
CD-Bonn	0.809	−0.233	−0.074	0.880	1.045	−0.327	−0.065	1.183	0.939	−0.313	−0.065	1.068
Argonne	0.743	−0.213	−0.075	0.801	0.953	−0.300	−0.065	1.073	0.852	−0.288	−0.068	0.963

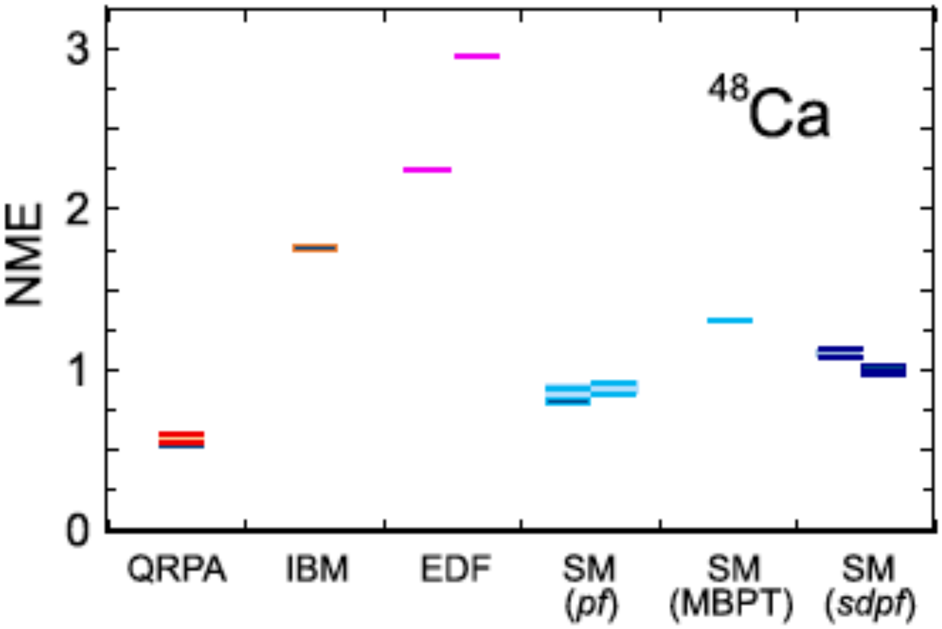
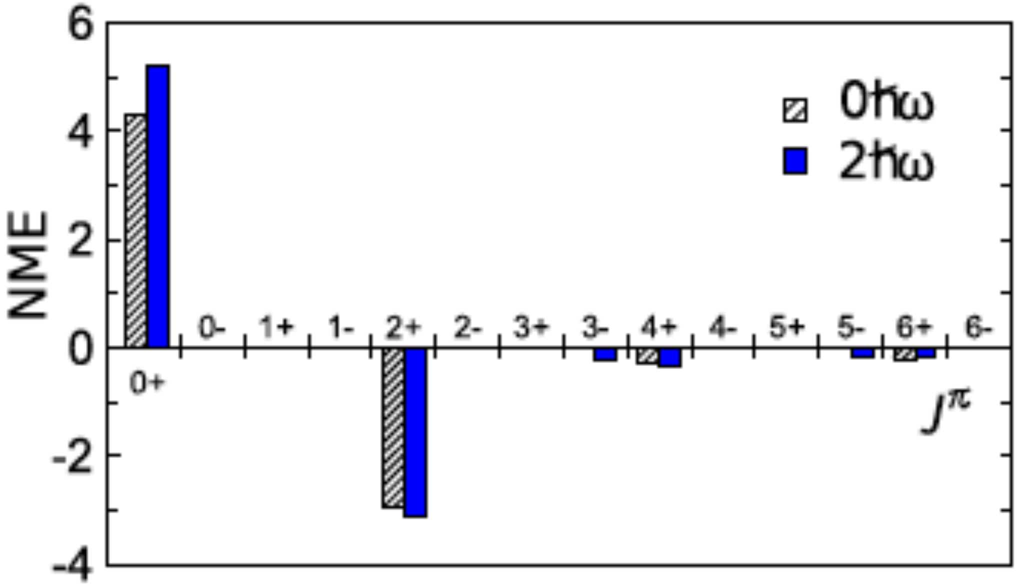


FIG. 2. Comparison of NME values for the ^{48}Ca $0\nu\beta\beta$ decay. The present shell-model results in the $sdpf$ space (SM $sdpf$: left SDPFMU-DB, right SDPFMU) are compared to pf-shell results (SM pf: left [17], right [15]), pf-shell result plus a perturbative calculation of the effect of orbitals outside the pf shell (SM MBPT) [50], QRPA [22], IBM [25], and EDF (left: nonrelativistic [26], right: relativistic [27]) calculations. The range between double horizontal bars covers results including a different type of short-range correlations (Argonne, CD-Bonn, UCOM [51]) and without them.

SM (pf)	[15] J. Menéndez, A. Poves, E. Caurier, and F. Nowacki, Nucl. Phys. **A818**, 139 (2009). [17] R. A. Sen'kov and M. Horoi, Phys. Rev. C **88**, 064312 (2013).
MBPT	[50] A. A. Kwiatkowski *et al.*, Phys. Rev. C **89**, 045502 (2014).
QRPA	[22] F. Šimkovic, V. Rodin, A. Faessler, and P. Vogel, Phys. Rev. C **87**, 045501 (2013).
IBM	[25] J. Barea, J. Kotila, and F. Iachello, Phys. Rev. C **91**, 034304 (2015).
EDF	[26] N. L. Vaquero, T. R. Rodríguez, and J. L. Egido, Phys. Rev. Lett. **111**, 142501 (2013). [27] J. M. Yao, L. S. Song, K. Hagino, P. Ring, and J. Meng, Phys. Rev. C **91**, 024316 (2015).

NME can be decomposed into individual contributions with spin/parity (J^π) of two decaying neutrons :

$$M^{0\nu} = \sum_J \langle 0_f^+ | \sum_{i\leq j,k\leq l} M^J_{ij,kl} [(\hat{a}_i^\dagger \hat{a}_j^\dagger)^J (\hat{a}_k \hat{a}_l)^J]^0 | 0_i^+ \rangle$$



What can be physical implications ?

What can be foreseen ?

Outline

1. Neutrinoless double beta decay ^{48}Ca -> ^{48}Ti
2. Decay from neutron closed-shell nuclei
3. Decay from spherical open-shell nuclei
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The model space is truncated to the f $_{7/2}$ only, for the purpose of schematic studies of certain basic features.

^{48}Ca ground state => f $_{7/2}$ closed shell

This is also the condensate of $J=0^+$ pair (4 pairs).

We can take $f_{7/2}$ part of the dbd operator.

$$M^{0\nu} = \sum_J \langle 0_f^+ | \sum_{i\leq j,k\leq l} M^J_{ij,kl} [(\hat{a}_i^\dagger \hat{a}_j^\dagger)^J (\hat{a}_k \hat{a}_l)^J]^0 | 0_i^+ \rangle$$

$$i, j, k, l = \mathrm{f}_{7/2}$$

$f_{7/2}$ single orbit model

^{48}Ca ground state : $(S^+)^4 |0>$ closed shell

$S^+_{n\,(p)}$: J=0 pair of neutrons (protons)

$D^+_{n\,(p)}$: J=2 pair of neutrons (protons)

Neutrinoless double beta decay operator $x_0\, S^+_p\, S_n + x_2\, D^+_p\, D_n + \ldots$

$x_0=-3.32$, $x_2=-2.37$
same sign

$S^+_p\, S_n\, (S^+_n)^4 |0> \;\sim + S^+_p\, (S^+_n)^3 |0>$

$D^+_p\, D_n\, (S^+_n)^4 |0> \;\sim - D^+_p\, D^+_n\, (S^+_n)^2 |0>$

because $D_n\, (S^+_n)^4 |0> \;\sim - D^+_n\, (S^+_n)^2 |0>$

opposite sign

Quadrupole-quadrupole proton-neutron interaction gives ^{48}Ti ground state as

$(c_0\, S^+_p\, S^+_n \; + c_2\, D^+_p\, D^+_n)\, (S^+_n)^2 |0>$

with c_0 and c_2 both positive

Thus, there is always cancellation between $S^+_p\, S_n$ and $D^+_p\, D_n$ contributions.

Monopole Pairing interaction V_M , T=1 J=0 part of the interaction.

$$V_M = - G_0 \ (½) \ ([a^+_j \ a^+_j \ ]^{(0)} \ [a\sim_j \ a\sim_j \ ]^{(0)} \ )$$
$$= - G_0 \ \ (S^+ \ S \sim)$$

where $G_0 = - <j^2; J=0 \mid V \mid j^2; J=0 >$ usually $G_0 > 0$ (2.24 for GXPF1B)

Quadrupole Pairing interaction V_Q , T=1 J=2 part of the interaction.

$$V_Q = - G_2 \ (½) \ ([a^+_j \ a^+_j \ ]^{(2)} \ [a\sim_j \ a\sim_j \ ]^{(2)} \ )$$
$$= - G_2 \ \ (D^+ \ D \sim)$$

where $G_2 = - <j^2; J=2 \mid V \mid j^2; J=2 >$ usually $G_2 > 0$ (0.94 for GXPF1B)

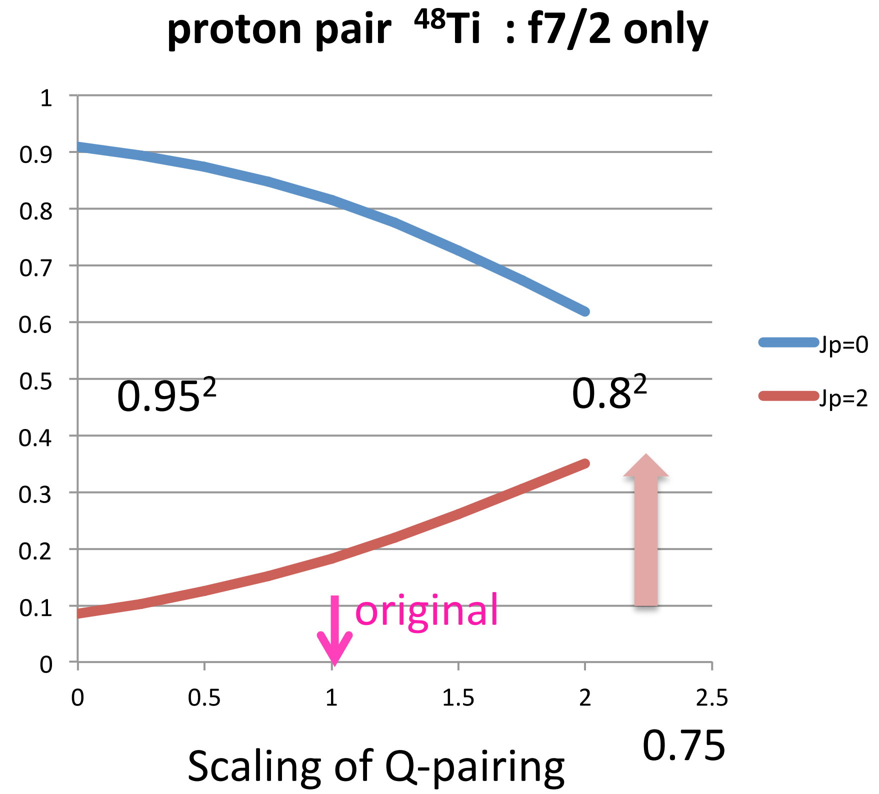
We can verify some properties,
by tuning the strength of the quadrupole pairing (Q-pairing) interaction V_Q.

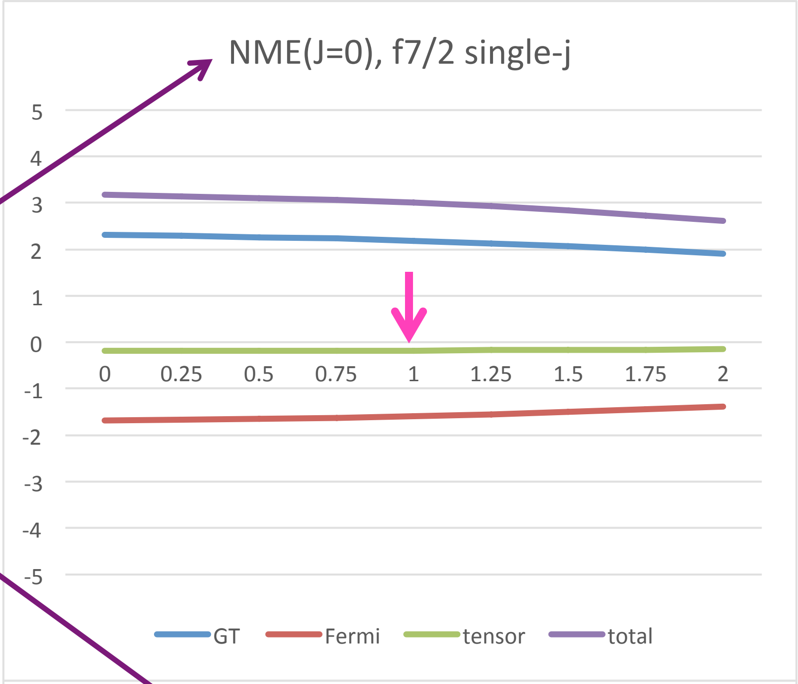
The ground state of ^{48}Ti

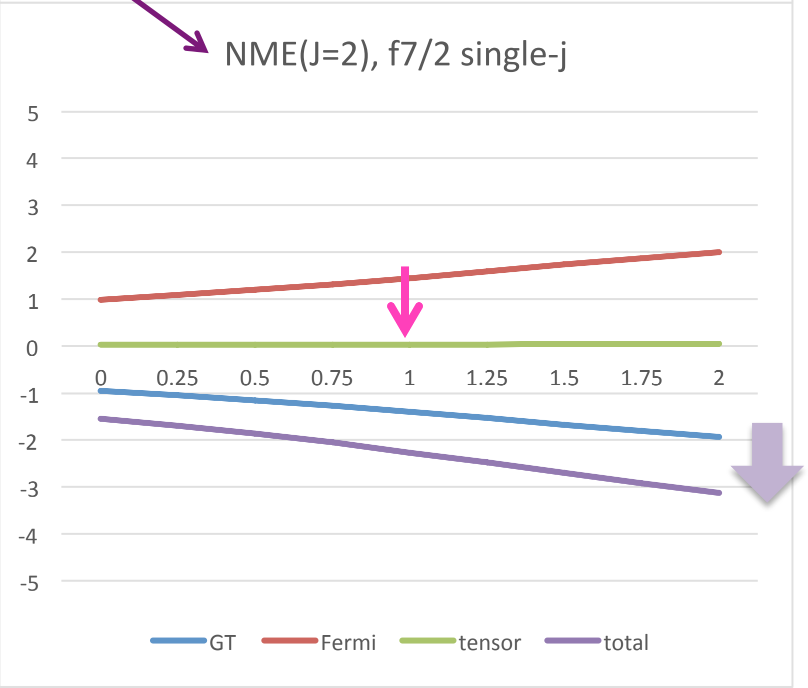
$$=\{(c_0\, S^{+}_{p}\, S^{+}_{n} + \underbrace{c_2\, D^{+}_{p}\, D^{+}_{n})}\, (S^{+}_{n})^2 + \ldots\}\ |0>$$

of the proton pair = this probability

⇨ change of NME by J=0 and 2





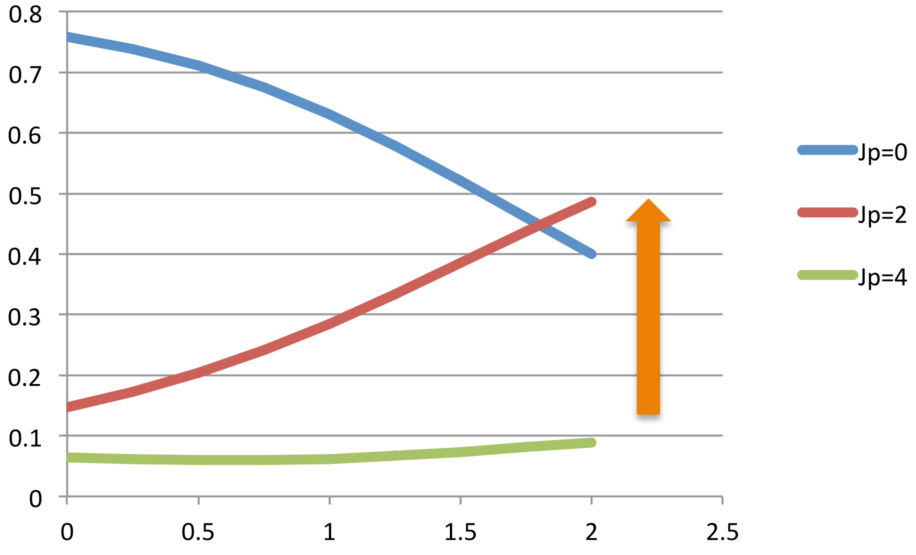


This feature can be seen in the full pf shell calculation

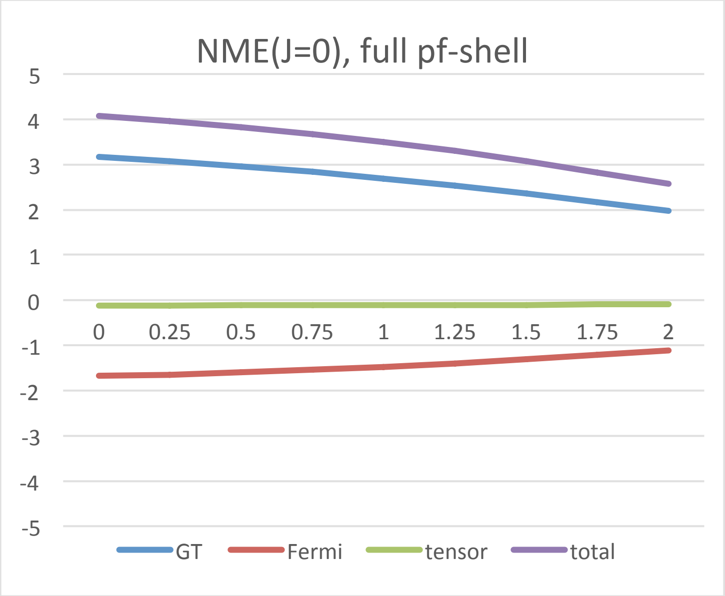
$$\left.\begin{aligned} S\ \text{pair} &= \Sigma_i\, \alpha_i\, S_i \\ D\ \text{pair} &= \Sigma_{i<i}\, \beta_{ij}\, D_{ij} \end{aligned}\right\}\ \textit{coherent sum}$$

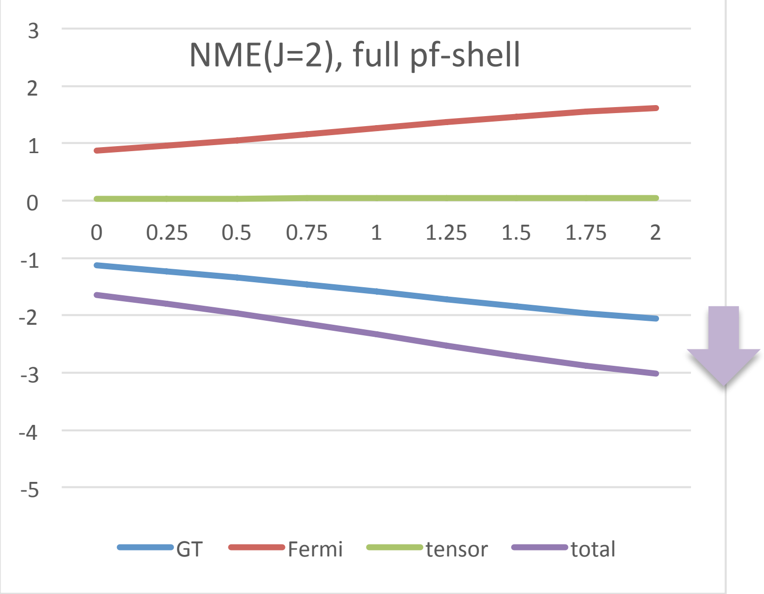
The J=2 T=1 TBME's are scaled.

proton pair ^{48}Ti : pf shell



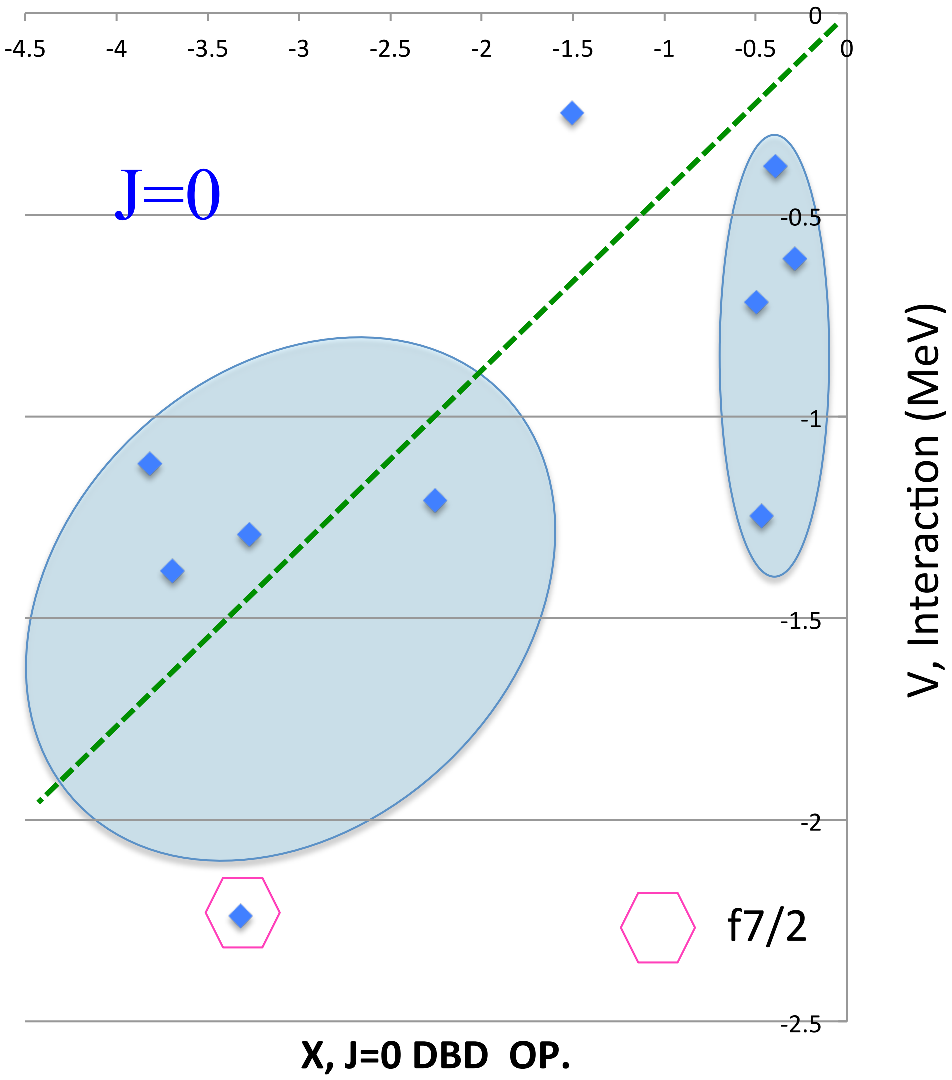
Scaling of Q-pairing

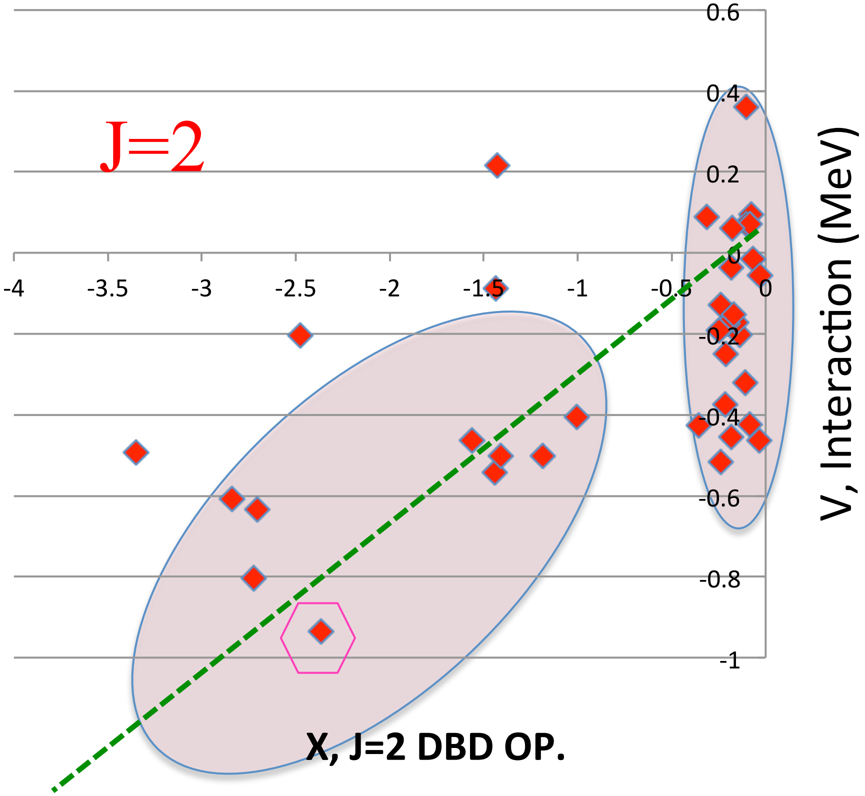

NME(J=0), full pf-shell


NME(J=2), full pf-shell

Correlation between TBME's of Interaction and double-beta decay

< i j ; J |V or X | k l ; J >





Two categories
(a) correlations
(b) small values for dbd

Signs are basically the same

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Spherical ground state in open shell

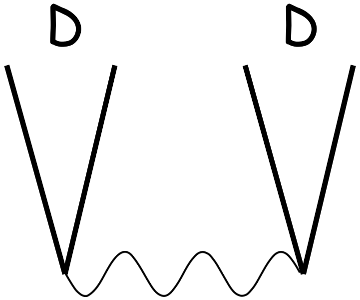
ground state : $(S^{+})^{k}\,|0>$ condensation of J=0 pairs

Quadrupole-pairing neutron-neutron interaction mixes as

$c_1\,(S^{+}{}_{n})^{k} - c_2\,(D^{+}{}_{n}\,D^{+}{}_{n})\,(S^{+}{}_{n})^{k-2}\;|0>$

with c_1 and c_2 both positive.

Ground state correlation (backward scattering) amplitude.



amplitudes with opposite sign between
neutron-neutron Q-pairing interaction
and
neutron-neutron quadrupole-quadrupole interaction (which is much weaker)

This is a robust property determined by the double commutator.

Quadrupole-pairing neutron-neutron interaction mixes as

$c_1\,(S^+_n)^k - c_2\,(D^+_n\,D^+_n)\,(S^+_n)^{k-2}\ \ |0>$

with c_1 and c_2 both positive.

Double-beta decay from this state tends to lead to

$x_1'\,S^+_p\,(S^+_n)^{k-1} - x_2'\,(D^+_p\,D^+_n)\,(S^+_n)^{k-2}\ \ |0>$

with x_1' and x_2' both positive

Another source of cancellation

Quadrupole-quadrupole proton-neutron interaction mixes as

$(c_0\,S^+_p\,S^+_n\ + c_2\,D^+_p\,D^+_n)\,(S^+_n)^2\,|0>$

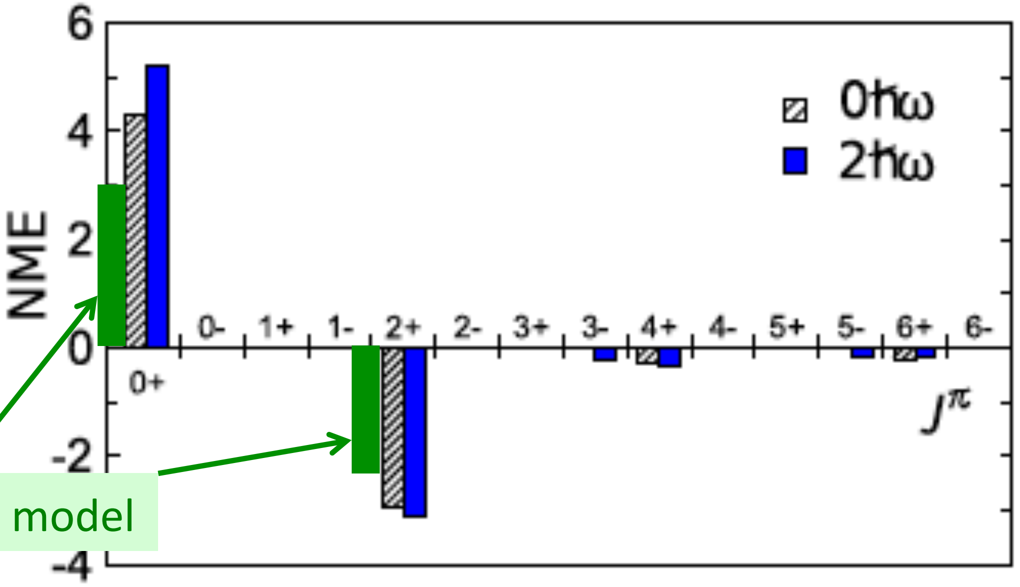
with c_0 and c_2 both positive

assuming Q-pairing J=2 pair ~ $[\,Q, S^+\,]$ ~ J=2 dbd pair

M-pairing J =0 pair ~ J=0 dbd pair

Contributions to NME increase in magnitude :

$$M^{0\nu} = \sum_J \langle 0_f^+ | \sum_{i\le j,k\le l} M^J_{ij,kl} [(\hat{a}_i^\dagger \hat{a}_j^\dagger)^J (\hat{a}_k \hat{a}_l)^J]^0 | 0_i^+ \rangle$$



single f 7/2 model

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Rotational nuclei

Intrinsic state of the ground-state rotational band

Particle-number projected HFB state

$$(\Lambda^{+}_{p})^{Kp}\ (\Lambda^{+}_{n})^{Kn}\ |0>$$

? Double beta decay

$$(\Lambda^{+}_{p})^{Kp+1}\ (\Lambda^{+}_{n})^{Kn-1}\ |0>$$

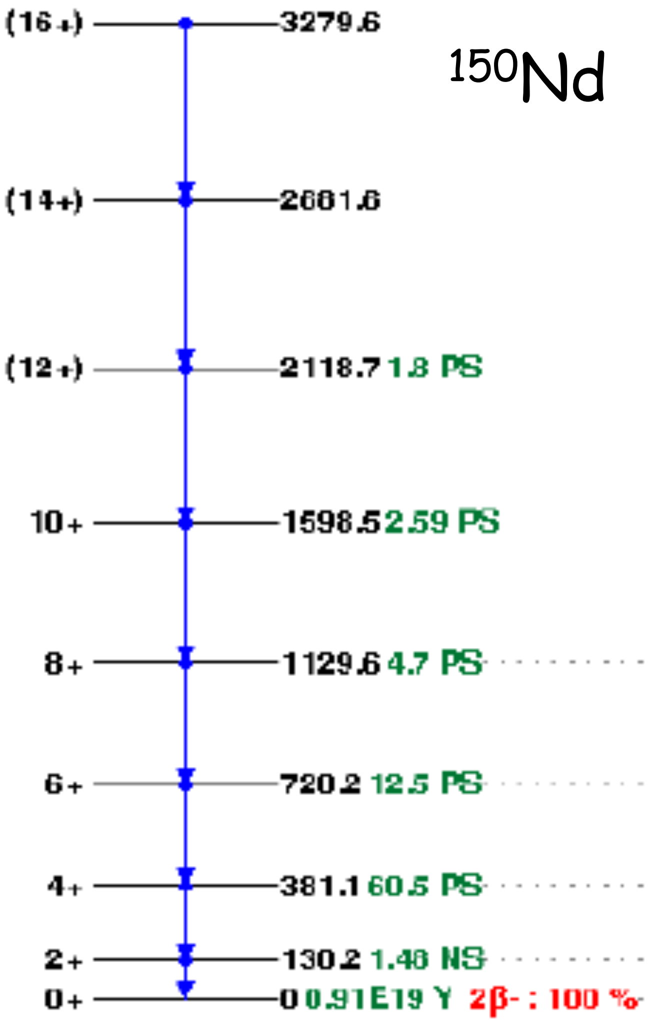
deformed pair : $\Lambda^{+}_{p,n} = \xi_0\, S^{+}_{p,n} + \xi_2\, D^{+}_{p,n} + \xi_4\, G^{+}_{p,n} + \ldots$

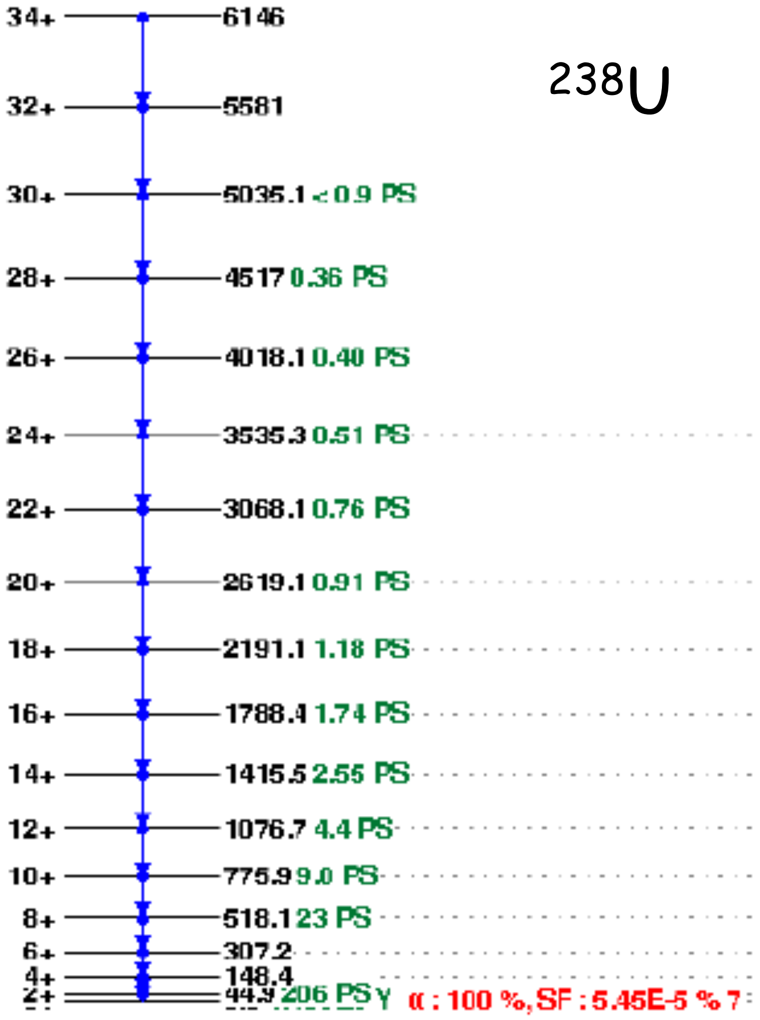
Higher order terms may contribute, but HFB with *N* and J^p projections will include all effects.

Possibility of a larger NME … good for establishing 0νν dbd

Candidates ^{150}Nd, ^{238}U

From the standard list ^{48}Ca、^{76}Ge、^{82}Se、^{96}Zr、^{100}Mo、^{116}Cd、^{128}Te、^{130}Te、^{150}Nd、^{238}U





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