Nuclear Structure and $\beta\beta$ Decay;
A Shell Model View

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OUTLINE

- The Interaction and its Parts
- The Many Body Methods; which parts of the interaction do they see
- What components of the WF's do the NME's explore?
- The origin of the discrepancies in the NME's
- Do we need to quench the 0ν operator?
- Conclusions.

The Interaction and its Parts

The Spherical Mean Field

L=0 Isovector and Isoscalar Pairing

$(Q^{\lambda} \cdot Q^{\lambda})$, mainly λ=2,3,4.

L=2 Isovector Pairing (?)

The many body methods; which parts of the Interaction do they see

- SM-CI, all of them
- QRPA, L=0 Isovector Pairing, λ (vibrations)
- SCCM, L=0 Isovector Pairing, λ (vibrations + permanent deformation)
- IBM, L=0 Isovector Pairing, λ=2 (vibrations + permanent deformation)
- I assume that all the methods take care properly of the Spherical Mean Field (not applies to IBM)

The Double Beta Decay process

This process exists due to the nuclear pairing interaction that favors energetically the even-even isobars over the odd-odd ones.

A nucleus is a potential $\beta\beta$ emitter just by accident. Thus, there cannot be systematic (experimental) studies in this field. One has to take what Nature gives

The neutrinoless double beta decay

The expression for the neutrinoless beta decay half-life, in the mass mode, for the $0^+ \rightarrow 0^+$ decay, can be brought to the following form:

$$[T_{1/2}^{(0\nu)}(0^+ - > 0^+)]^{-1} = G_{0\nu}\left(M^{(0\nu)}\left(\frac{\langle m_\nu \rangle}{m_e}\right)\right)^2$$

$G_{0\nu}$ is the kinematic phase space factor, $M^{0\nu}$ the nuclear matrix element (NME) that has Fermi, Gamow-Teller and Tensor contributions, and $\langle m_\nu \rangle$ the effective neutrino mass.

The neutrinoless double beta decay

$$M^{(0\nu)} = \left(\frac{g_A}{1.25}\right)^2 \left(M_{GT}^{(0\nu)} - \frac{M_F^{(0\nu)}}{g_A^2} - M_T^{(0\nu)} \right)$$

$$\langle m_\nu \rangle = \sum_k U_{ek}^2 m_k$$

The U's are the matrix elements of the weak mixing matrix.

The Nuclear Matrix Elements

The matrix elements $\mathrm{M}^{(0\nu)}_{GT,F,T}$ can be written as,

$$M^{(0\nu)}_{K} = \langle 0^+_f | H_K(|\vec{r_1} - \vec{r_2}|)(t_1^- \ t_2^-)\Omega_K | 0^+_i \rangle$$

$$\text{with} \ \ \Omega_F = 1, \ \Omega_{GT} = \vec{\sigma}_1 \cdot \vec{\sigma}_2, \ \Omega_T = S_{12}$$

$H_K(|\vec{r_1} - \vec{r_2}|)$ **are the neutrino potentials (∼1/r) obtained from the neutrino propagator.**

The Nuclear Matrix Elements

The neutrino potentials have the following form:

$$H_K^m(r_{12}) = \frac{2}{\pi g_A^2} R \int_0^\infty f_K(qr_{12}) \frac{h_K(q^2) q dq}{q + E_m - (E_i + E_f)/2}$$

$h_F(q^2) = g_V(q^2)$ **and, neglecting higher order terms in the nuclear current,** $h_{GT}(q^2) = g_A(q^2)$ **and** $h_T(q^2) = 0$.

The energy of the virtual neutrino (q) is about 150 MeV. Therefore, to a very good approximation, E$_m$ can be replaced by an average value. This is the closure approximation.

The 0ν operators; Consensus

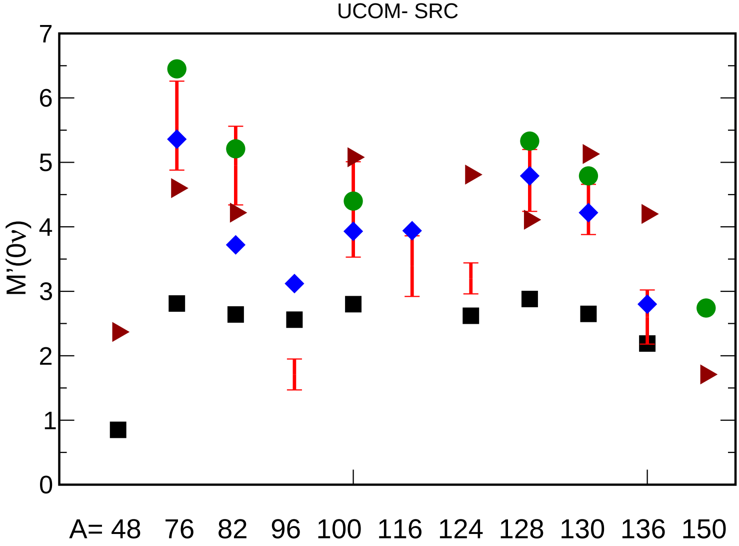
There is a broad consensus in the community about the form of the transition operator in the mass mode,

- It must include higher order terms in the nuclear current,
- And the proper nucleon dipole form factors, isovector and isoscalar.
- The consensus extends to the validity of the closure approximation for the calculation of the NME's
- And to the use of very soft short range corrections.

The NME's of the 0ν operator. They are sensitive to what and how?

- When Isovector pairing is dominant in both nuclei (superfluid limit) the NME's are very large
- $(Q^\lambda \cdot Q^\lambda)$ correlations REDUCE the NME's
- Isocalar pairing seems to REDUCE the NME's as well
- In general, any STRUCTURAL difference between the initial and final nucleus tends to REDUCE the NME

In fact, the dispersion of the NME's is still too large



QRPA(Tu) (bars) QRPA(Jy)(lozenges) IBM(circles) ISM(squares) GCM(triangles)

The Nuclear Wave Functions

To assess the validity of the wave functions, quality indicators are needed based upon:

- The spectroscopy of the intervening nuclei
- The occupancies of the orbits around the Fermi level.
- The GT-strengths and strength functions, The 2ν matrix elements, etc.

This quality control should be applied on a decay by decay basis, because a given approach may work well for some cases and not for others.

Remember these facts when evaluating the different approachs

- In general, any structural difference between the initial and final nucleus tends to REDUCE the NME
- When Isovector pairing is dominant in both nuclei (superfluid limit) the NME's are very large
- Isocalar pairing REDUCES the NME's
- $(\mathbf{Q}^{\lambda} \cdot \mathbf{Q}^{\lambda})$ correlations REDUCE the NME's as well

The NME's and the mismatch of the WF's

A very spectacular example of the cancellation of the NME by the mismatch of the WF's is provided by the ^{48}Ca decay. The seniority structures of the two nuclei are very different.

^{48}Ca, ν=0, 97%, ν=4, 3%
^{48}Ti, ν=0, 59%, ν=4, 36% ν=6, 4%, ν=8, 1%

The matrix elements $\langle \nu_f(\beta)|O_{GT}|\nu_i(\alpha)\rangle$ are gathered below. There are two large matrix elements; one diagonal and another off-diagonal of the same size and opposite sign.

The NME's and the mismatch of the WF's

If the two nuclei were dominated by the seniority zero components one should obtain M$_{GT}$∼4. If ^{48}Ti were a bit more deformed, M$_{GT}$ will be essentially zero. The value produced by the KB3 interaction is 0.75 that is more than a factor five reduction with respect to the seniority zero limit.

^{48}**Ti**	$s = 0$	$s = 4$	$s = 6$	$s = 8$
^{48}**Ca** $s = 0$	**3.95**	**-3.68**	-	-
^{48}**Ca** $s = 4$	**0.00**	**-0.26**	**0.08**	**-0.02**

The Drift of the NME

When new QRPA calculations were made modifying the single particle energies as to reproduce the experimental occupancies, the NME's got reduced

The standard QRPA, IBM and GCM calculations violate badly isospin conservation

The consequence is an overestimation of the Fermi contribution to the NME

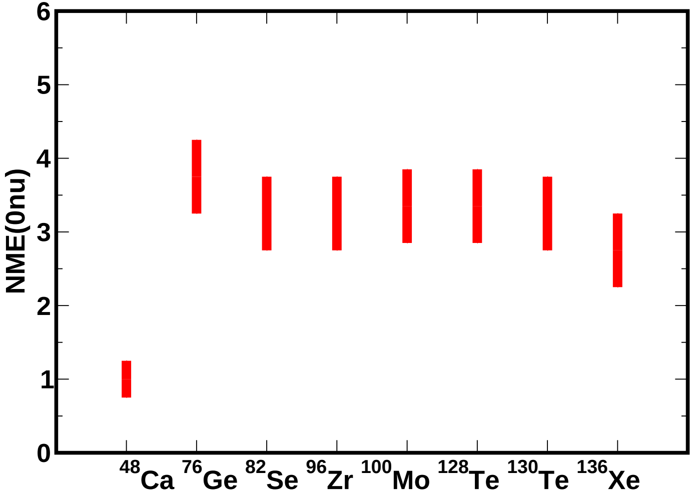
When isospin is restored the NME's are reduced typically a 20%

When the isoscalar pairing channel of the NN interaction is properly taken into account the NME's are reduced as well

Approaching consensus

- In view of all this arguments, one can surmise that the QRPA, IBM and GCM tend to overestimate the NME's
- On the other side, increasing the valence space of the ISM calculations tends to increase moderately the NME's
- Therefore, I dare to propose the following "safe" range of values (assuming no quenching)

A modest proposal . . .



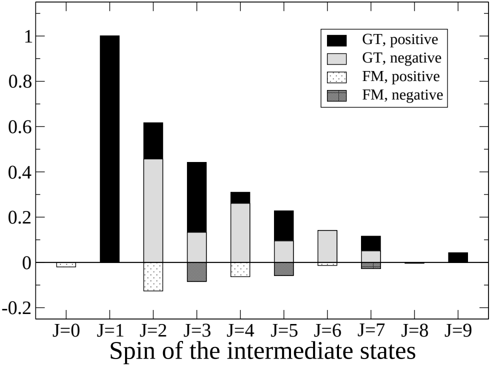
Quenching of the Gamow-Teller Strength

- The charge exchange experiments of the first generation only produced about one half of the Ikeda sum rule, and floods of ink have been spent in this problem
- And the GT single beta decays demand quenching factors ranging from 0.9 in the p-shell to 0.7 in heavy nuclei. It can be seen as the effective ”charge” for the GT operator,
- The missing strength problem is common to all the descriptions that use a basis of independent particles and regularized interactions

g_A, to quench (2ν), or not to quench (0ν)?

- To reproduce the experimental $2\nu\beta\beta$ lifetimes, it is compulsory to invoke the quenching factors discussed before
- We can distinguish between a secular quenching factor of 0.7 for calculations in complete major oscillator shells, and local quenching factors due to the limitations of the ISM valence spaces
- The open question is whether these quenching factors must be applied to the 0ν decays
- To be consistent with the closure approximation, the quenching factor must be the same for all the multipole channels. If not, each channel would require a separate treatment.

The contributions to the NME as a function of the J^π of the intermediate states: ^{82}Se → ^{82}Kr



R. A. Senkov, M. Horoi, and B. A. Brown, Phys. Rev. C 89, 054304

The origin of the Quenching of the Gamow-Teller Strength

- This is a very old topic; Is its origin "nucleonic" or non nucleonic?
- In modern language; Can we get it by doing standard MBPT on the $\vec{\sigma} \cdot \vec{\tau}$ operator?
- Or do we need to include two body currents?
- Probably both

Recent attempts to go beyond the standard approaches

- Menéndez, Gazit and Schwenk (2011) have studied the effect of two-body currents on single GT decays and on neutrinoless $\beta\beta$ decays using χEFT. They find that the quenching of the matrix elements of the GT decays is greater than that of the $0\nu\beta\beta$ NME's. In fact, the range of the modifications of the latter varies between +10% and –35% (corresponding to q(GT)=0.96 and q(GT)=0.74).
- One important open issue is what fraction of the standard quenching, q(GT)$\sim$0.7, is due to the two-body currents and which to many body purely nucleonic effects

Recent attempts to go beyond the standard approaches

- The many body renormalization of the $0\nu\beta\beta$r and $\vec{\sigma}\vec{\tau}$ operators, in a purely nucleonic description, has been recently addressed by Holt, Engel, Hagen and Navratil among others. Holt and Engel report an increase of 20-30% of the $0\nu\beta\beta$ NME's of ^{82}Se and ^{76}Ge respectively, correlated with values of q(GT) in the 0.85 range.
- This issue needs to be settled asap, but it seems that (if there is any) the quenching of g_A in the $0\nu\beta\beta$ decays is much smaller than in the $2\nu\beta\beta$ process

Conclusions

- There are solid arguments to submit that when quality controls are applied to the nuclear WF, the dispersion of the values of the NME's is very much reduced. That's good news
- Recent calculations of the effects of the Chiral two-body currents on the $0\nu\beta\beta$ and in the single GT beta decays show that the quenching factor of the latter cannot be directly translated into the former. More good news
- Many body PT shows that one can get a certain enhancement of the NME's due to purely nucleonic effects, while at the same time producing about half of the standard quenching. Even better!