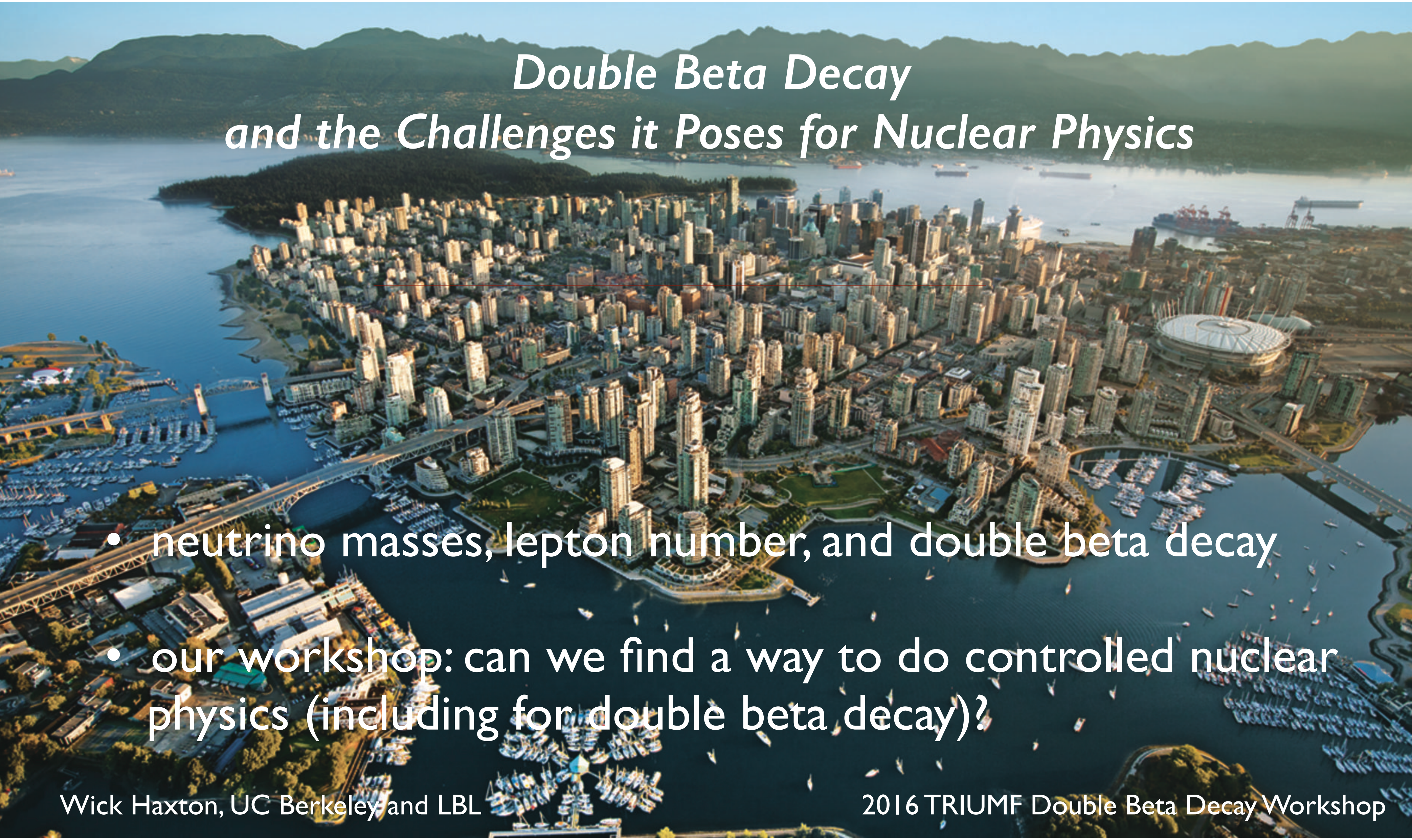


Double Beta Decay
and the Challenges it Poses for Nuclear Physics

- neutrino masses, lepton number, and double beta decay

- our workshop: can we find a way to do controlled nuclear physics (including for double beta decay)?

Wick Haxton, UC Berkeley and LBL

2016 TRIUMF Double Beta Decay Workshop

- 1934: Fermi's incorporation of both in his "effective theory" of β decay

$$n_{\text{bound}} \to p_{\text{bound}} + e^- + \bar{\nu}_e$$

- 1935: M. Goppert-Mayer describes "double β disintegration"

$$2n_{\text{bound}} \to 2p_{\text{bound}} + 2e^- + 2\bar{\nu}_e$$

- 1934: Fermi's incorporation of both in his "effective theory" of β decay

$$n_{\rm bound} \to p_{\rm bound} + e^- + \bar{\nu}_e$$

- 1935: M. Goppert-Mayer describes "double β disintegration"

$$2n_{\rm bound} \to 2p_{\rm bound} + 2e^- + 2\bar{\nu}_e$$

NO LEPTONS LEPTON NUMBER 2-2=0

- 1934: Fermi's incorporation of both in his "effective theory" of β decay

$$n_{\text{bound}} \rightarrow p_{\text{bound}} + e^- + \bar{\nu}_e$$

- 1935: M. Goppert-Mayer describes "double β disintegration"

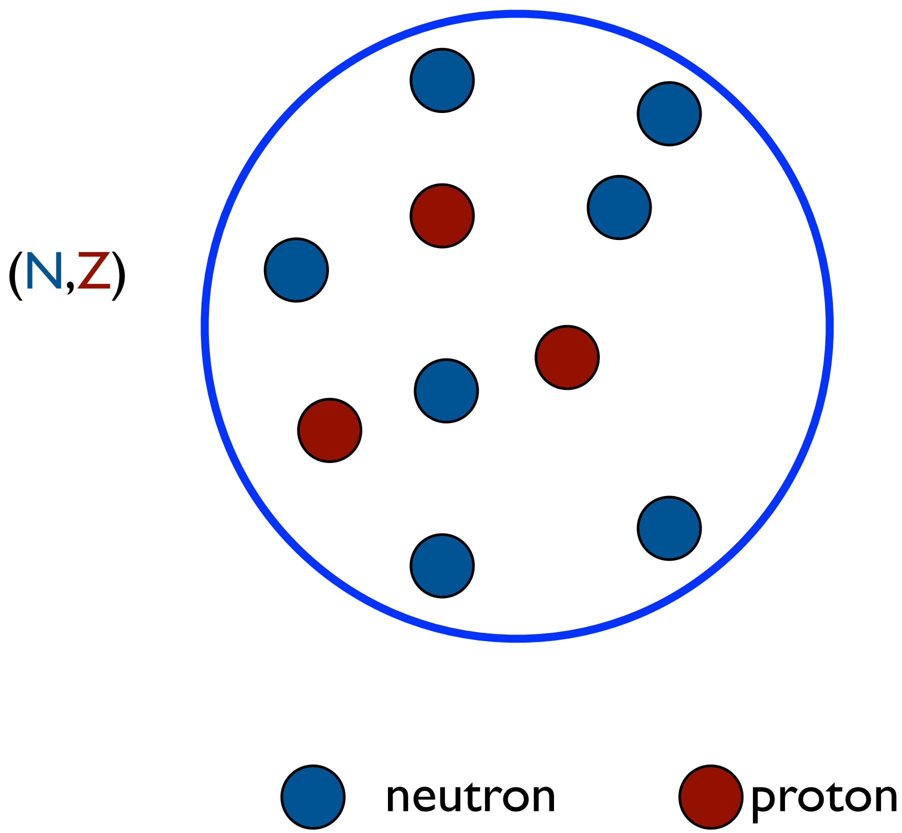
$$2n_{\text{bound}} \rightarrow 2p_{\text{bound}} + 2e^- + 2\bar{\nu}_e$$

- 1937: Majorana suggests that

$$\nu_e \equiv \bar{\nu}_e$$

In the same year Giulio Racah pointed out that Majorana's new theory would lead to a second form of ββ decay -- a neutrinoless type

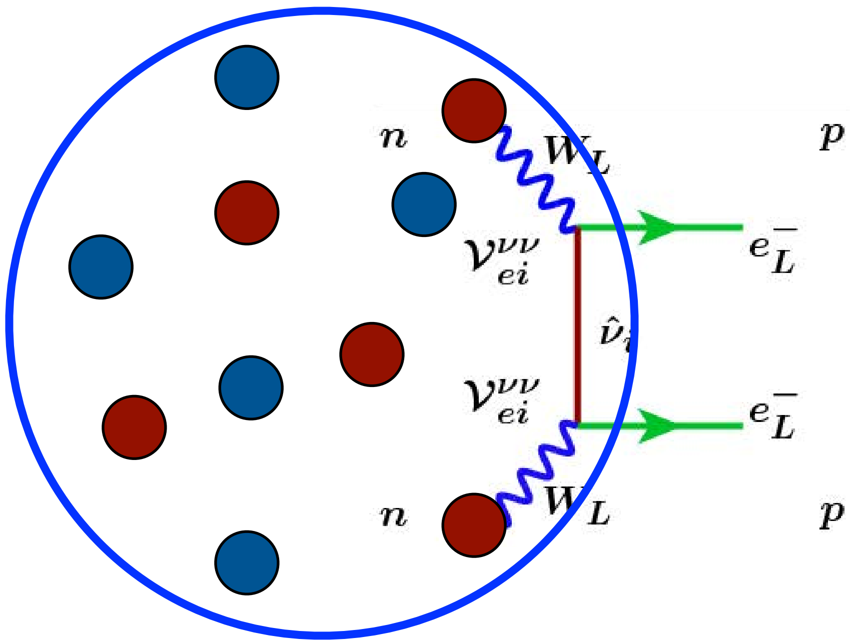
$$2n \rightarrow 2p + 2e^-$$



In the same year Giulio Racah pointed out that Majorana’s new theory would lead to a second form of ββ decay -- a neutrinoless type

$$2n \rightarrow 2p + 2e^-$$

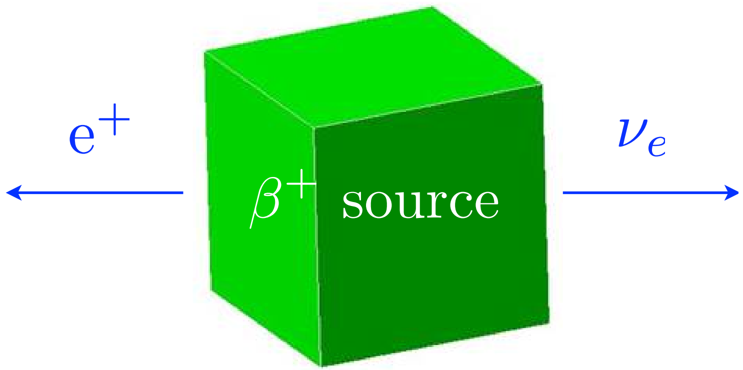
(N,Z)



neutron proton

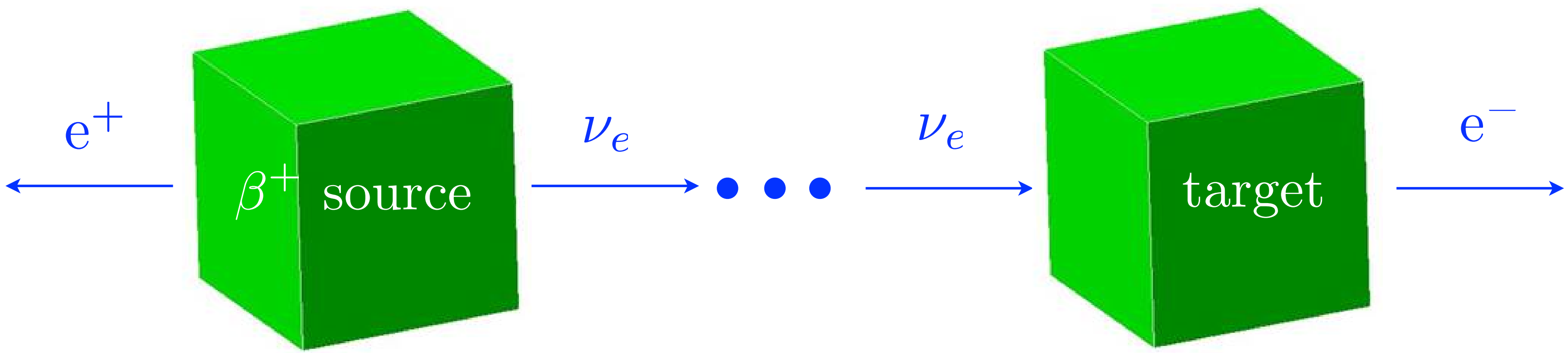
Can we do experiments to distinguish between the two kinds of neutrinos?

a “thought” experiment:



this defines the ν_e

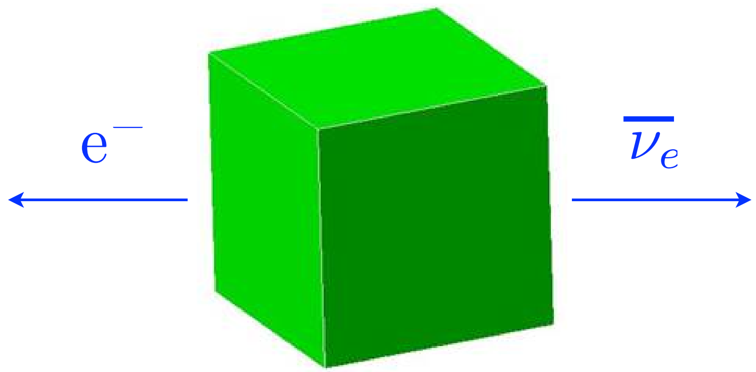
then allow it to interact in a target



this defines the ν_e

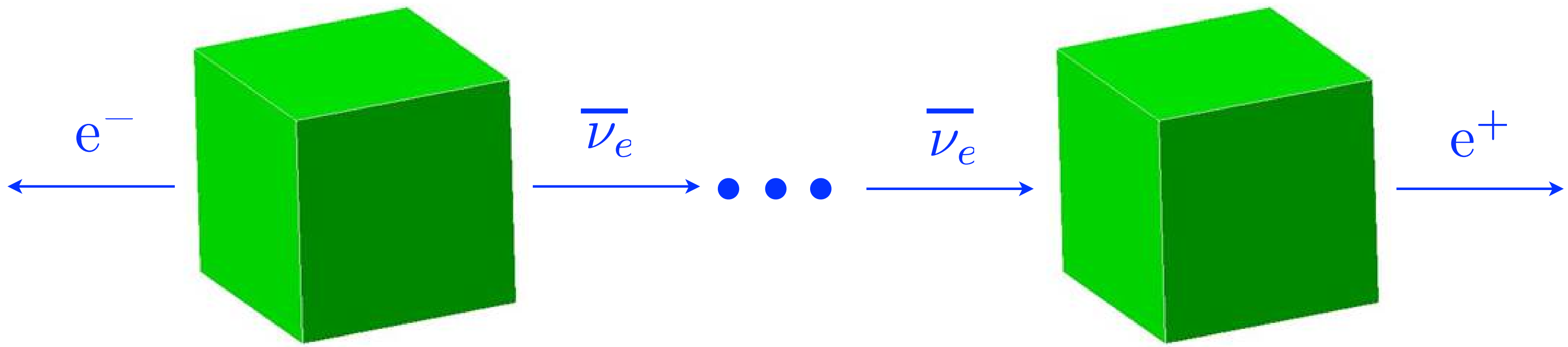
finding an e^- is produced

and then a second experiment



this defines the $\bar{\nu}_e$

allow it to interact in a target



this defines the $\bar{\nu}_e$

finding an e^+ is produced

So clearly (??) $|\nu_e\rangle \perp |\bar{\nu}_e\rangle$ not $|\nu_e\rangle \equiv |\bar{\nu}_e\rangle$

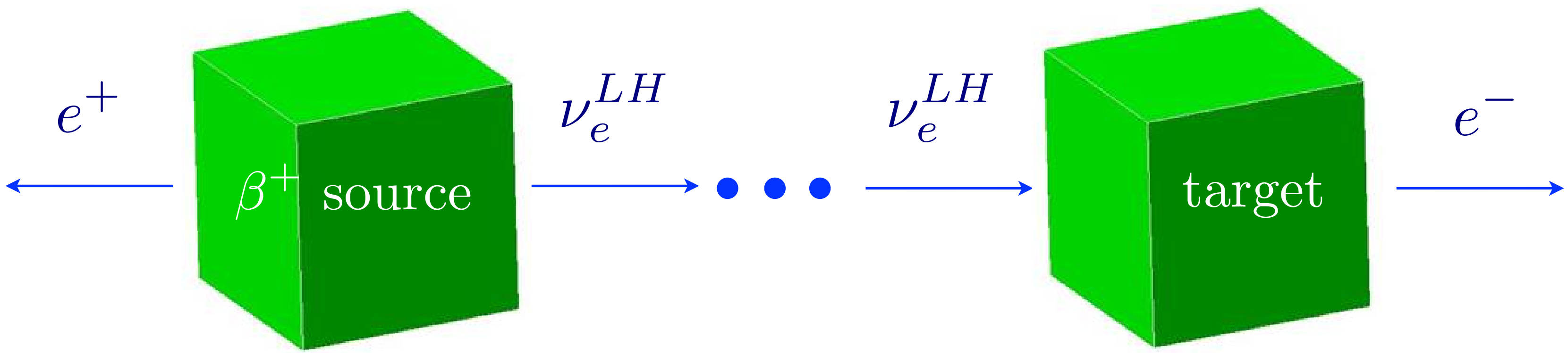
lepton	l_e
e^-	$+1$
e^+	-1
ν_e	$+1$
$\bar{\nu}_e$	-1

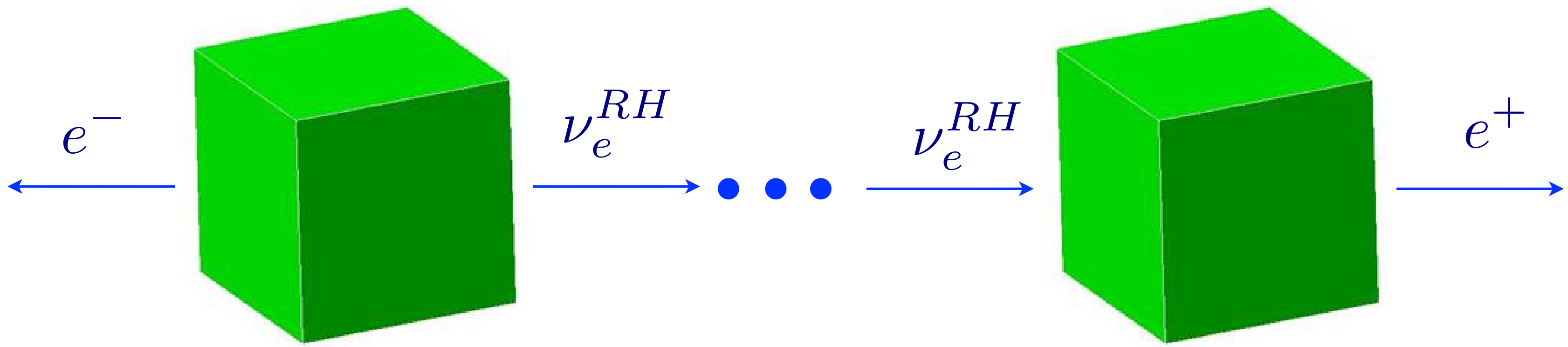
$$\sum_{\text{in}} \mathbf{l_e} = \sum_{\text{out}} \mathbf{l_e}$$

$$|\nu_e(l_e = 1)\rangle \neq |\bar{\nu}_e(l_e = -1)\rangle$$

The problem with this argument is PNC: the neutrino helicity

adding helicity labels

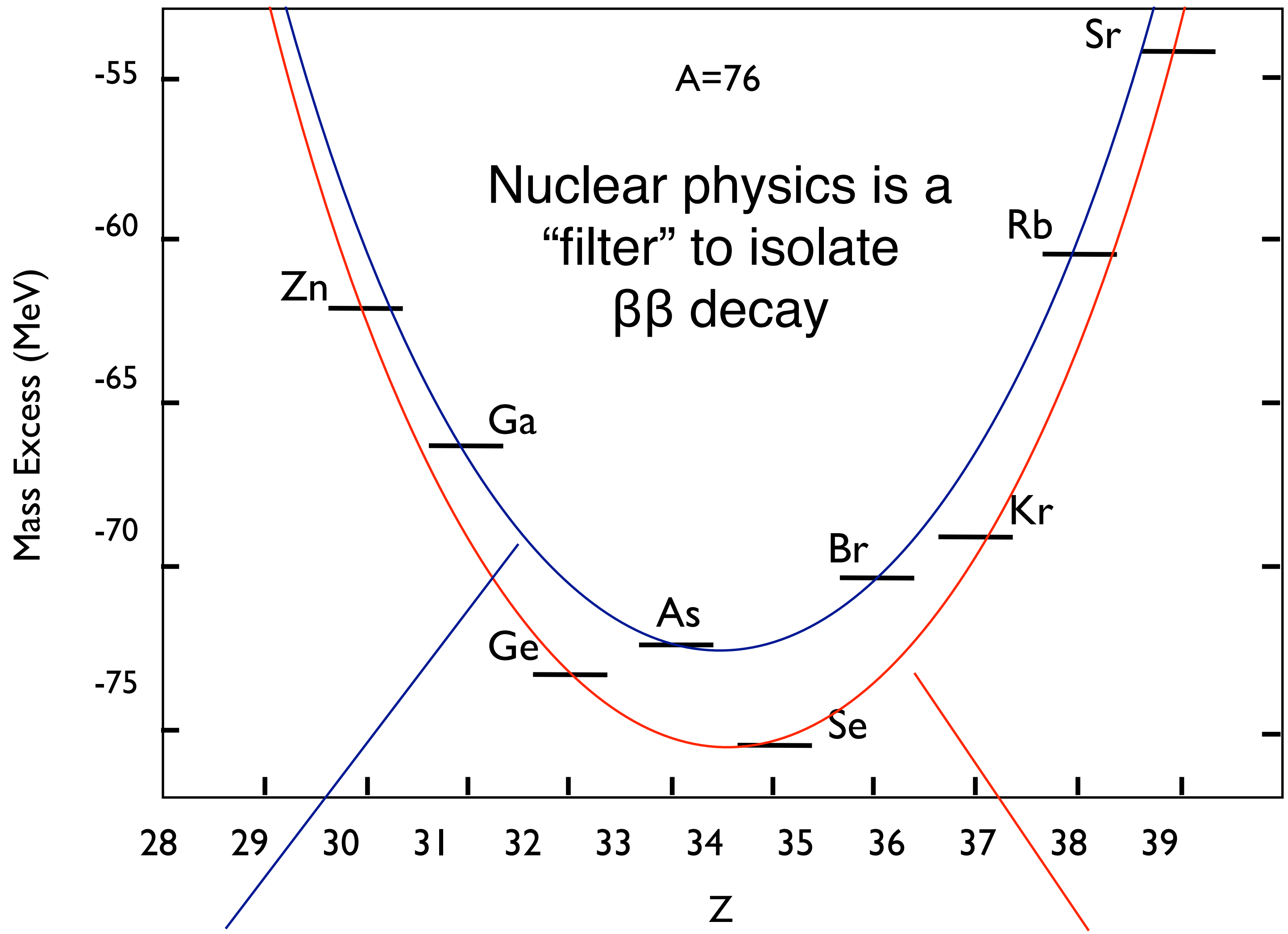




so no need to add any addition quantum number to account for observations

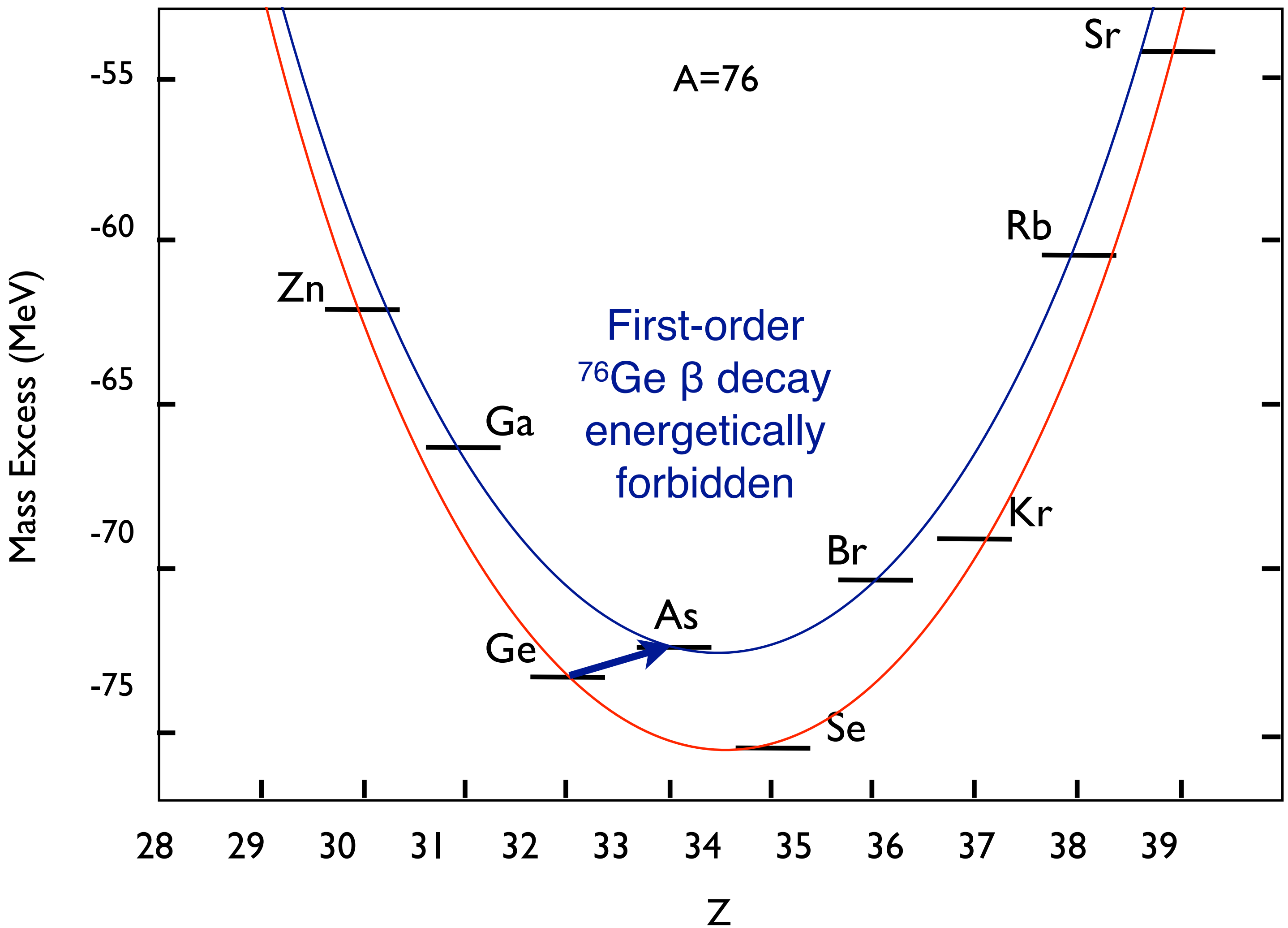
massless Dirac or Majorana neutrinos perform exactly as required

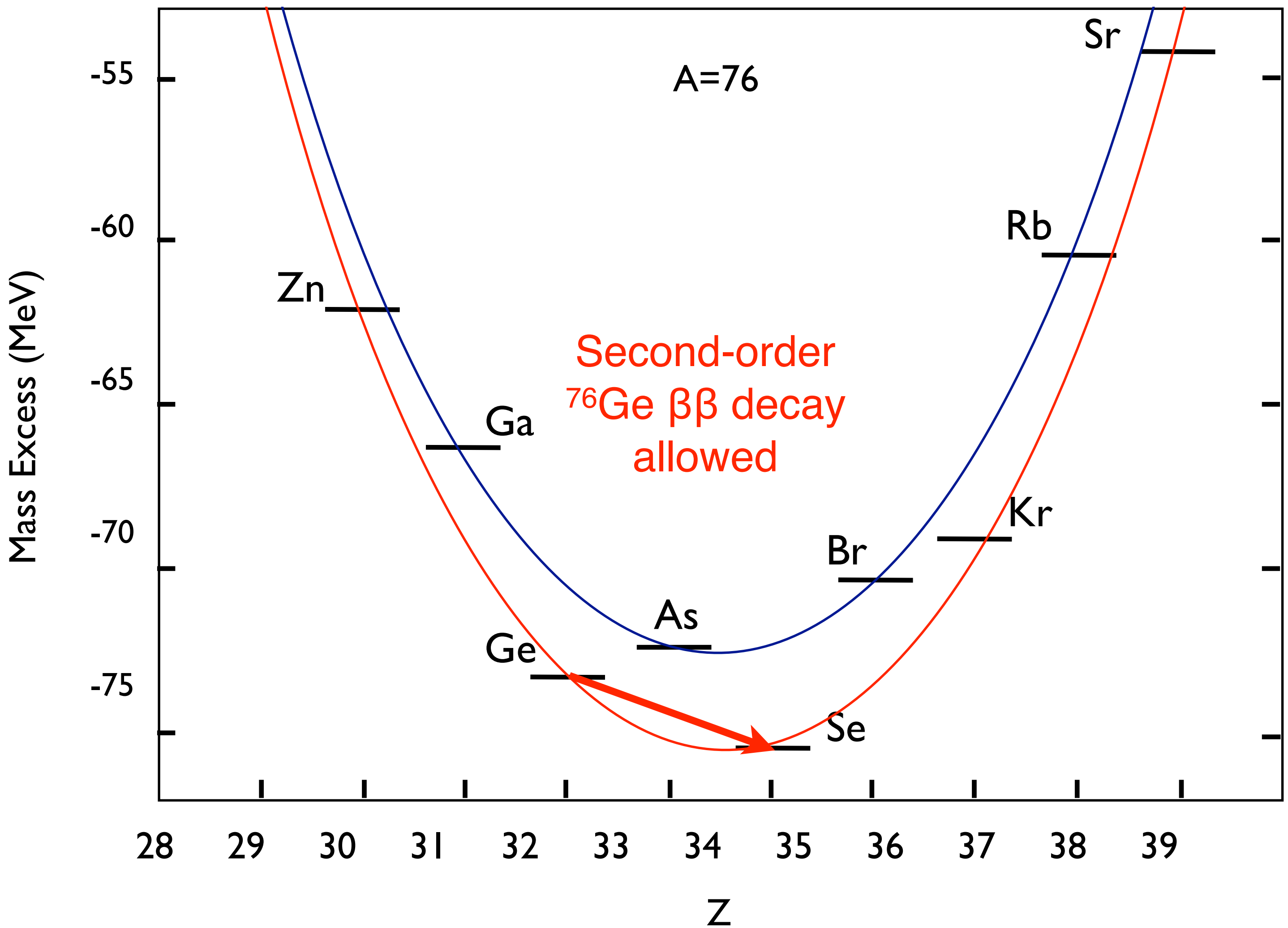
nuclear physics allows us to study these rare, 2nd order weak processes



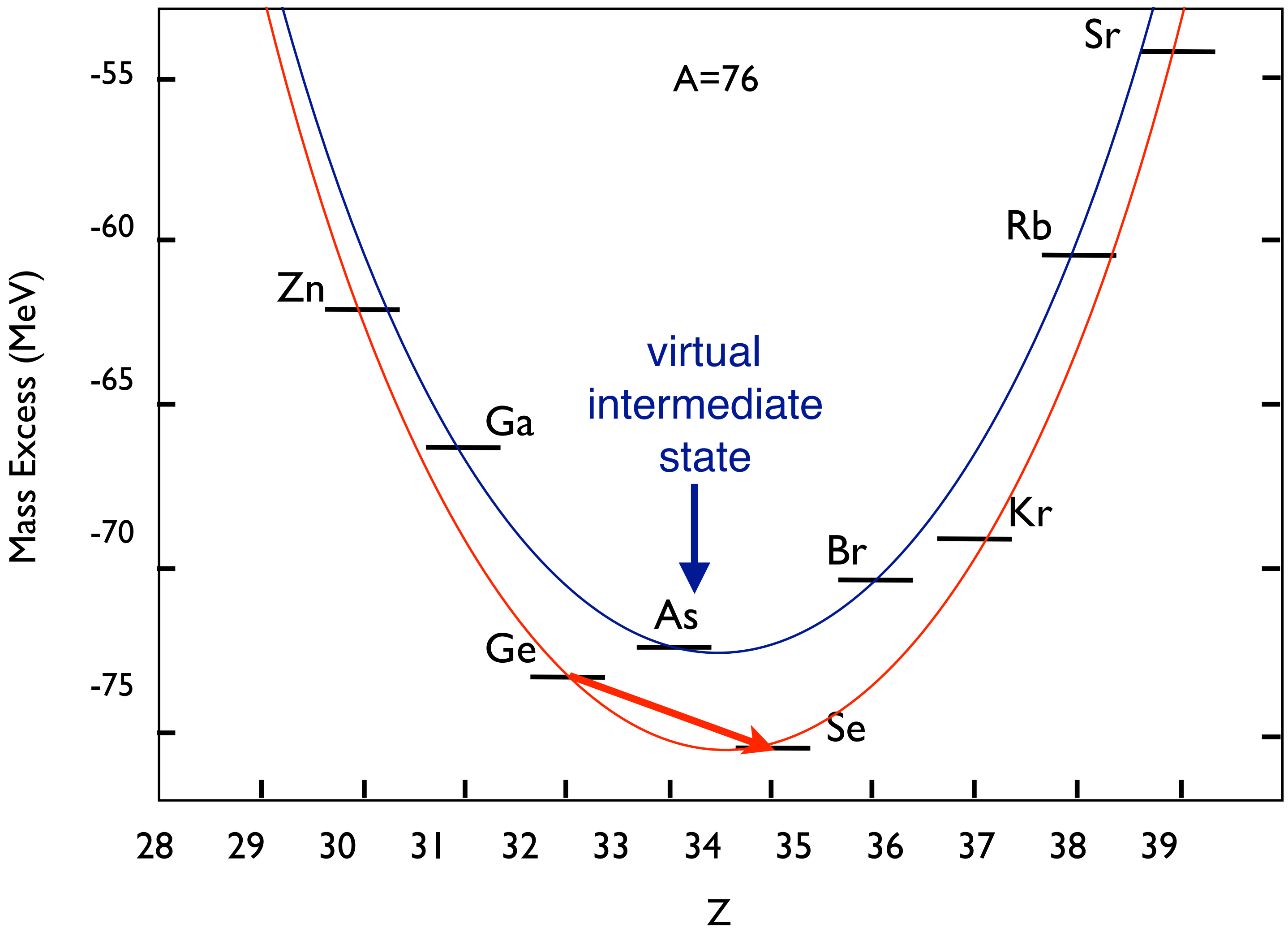
Odd N and Z nuclei:
two broken pairs

Even N and Z nuclei:
attractive pairing force

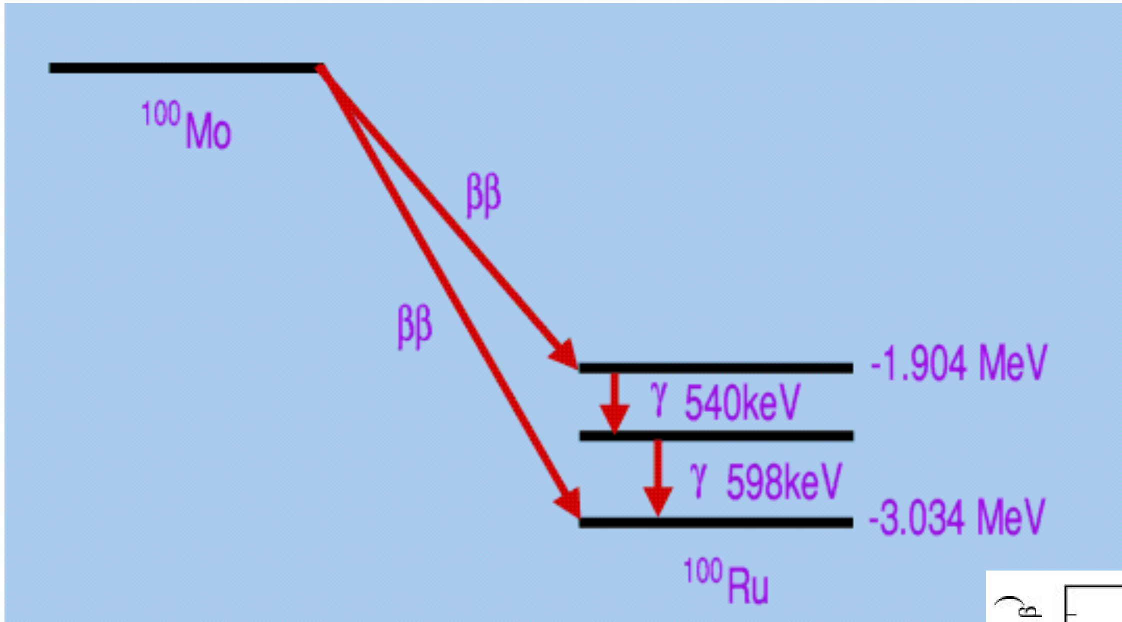

A=76
Sr
Rb
Zn
Ga
Kr
Br
As
Ge
Se
First-order
76Ge β decay
energetically
forbidden
-55
-60
-65
-70
-75
Mass Excess (MeV)
28
29
30
31
32
33
34
35
36
37
38
39
Z



About 50 cases where nuclear physics isolates very rare, second-order weak interactions



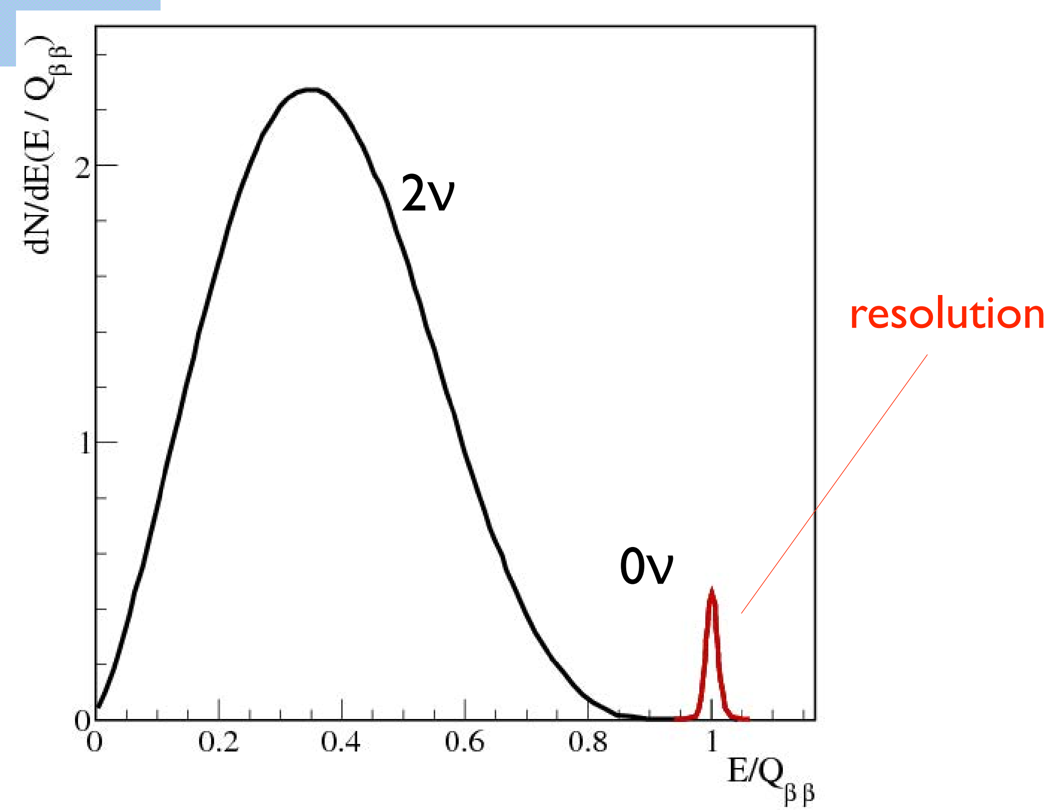
About 50 cases where nuclear physics isolates very rare, second-order weak interactions



The two ββ decay modes can be distinguished in experiments

spectrum of summed energy for the two outgoing electrons:

with good detector energy resolution, the 0ν and 2ν modes can be separated



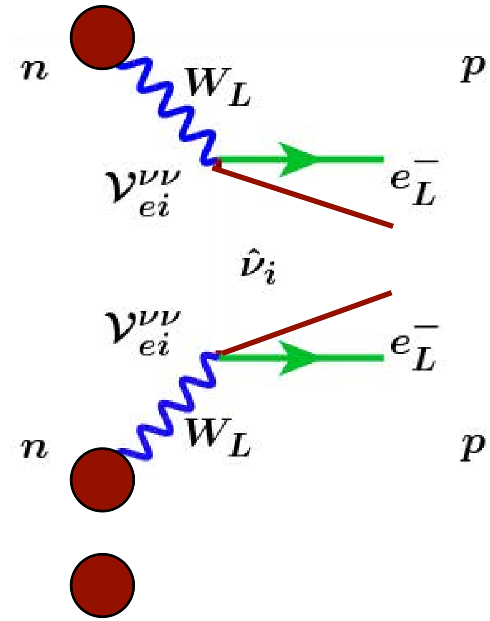
2ν ββ decay occurs regardless of whether $\nu = \bar{\nu}, \quad \nu \perp \bar{\nu}$

$$(N, Z) \rightarrow (N-1, Z+1) + e^- + \bar{\nu}_e$$

$$(N-1, Z+1) \rightarrow (N-2, Z+2) + e^- + \bar{\nu}_e \;\Rightarrow$$

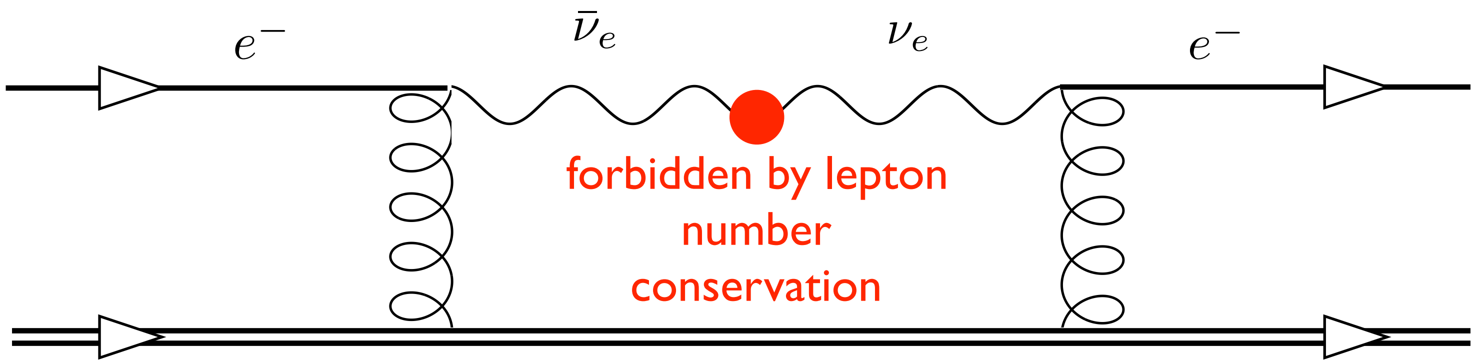
$$(N, Z) \rightarrow (N-2, Z+2) + 2e^- + 2\bar{\nu}_e$$

lepton-number conserving



what about neutrinoless ββ decay?

Forbidden if the neutrino is Dirac

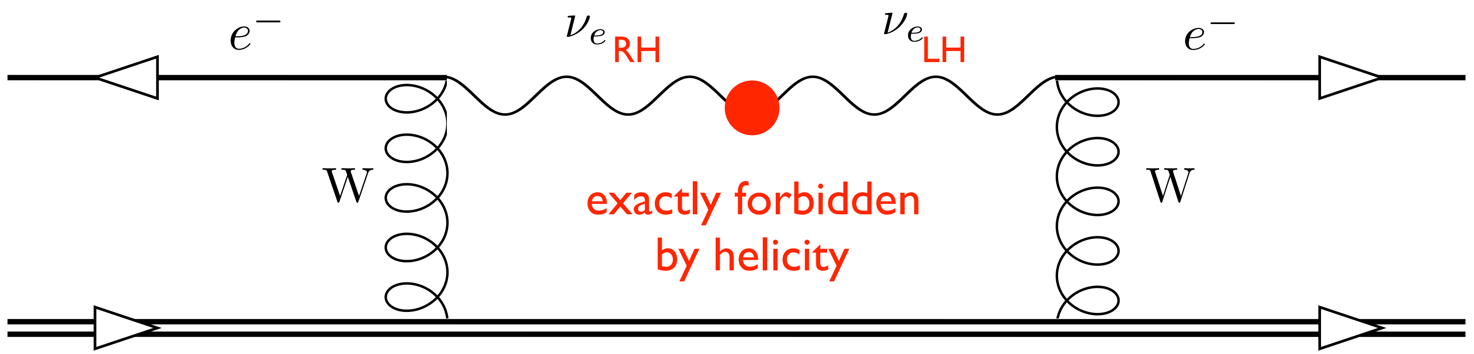


(which we concluded from our “thought” experiment showing $\nu \perp \bar{\nu}$)

Definitive it seems…. but in 1957 we discovered that weak interactions violated parity maximally

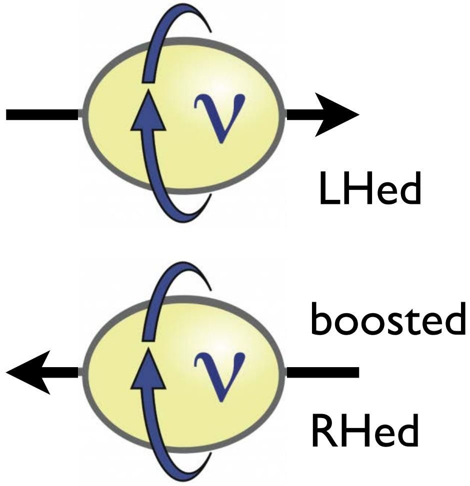
There is ANOTHER selection rule forbidding ββ decay - helicity

Forbidden for massless neutrinos that are purely handed



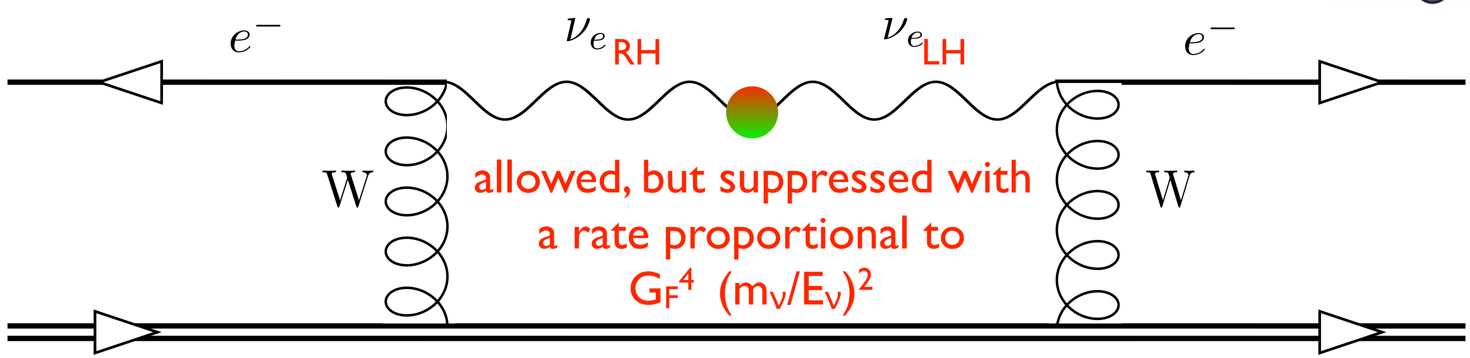
The good news: the interesting possibility that neutrinos are Dirac, Majorana, or both is in fact completely open

The bad news: we can't use 0ν ββ decay to decide this issue because helicity forbids the process, regardless



But now that we know neutrinos are massive:

If neutrinos have mass, helicity is not a particle label:
it can be reversed by jumping to a moving frame

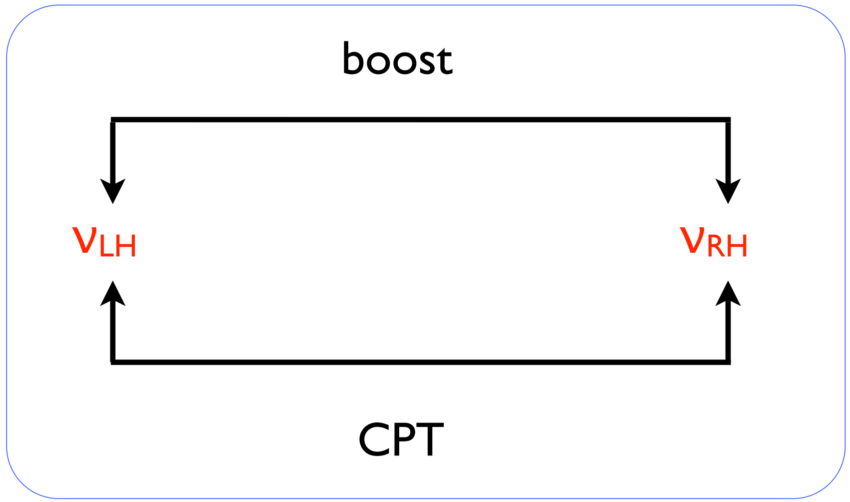


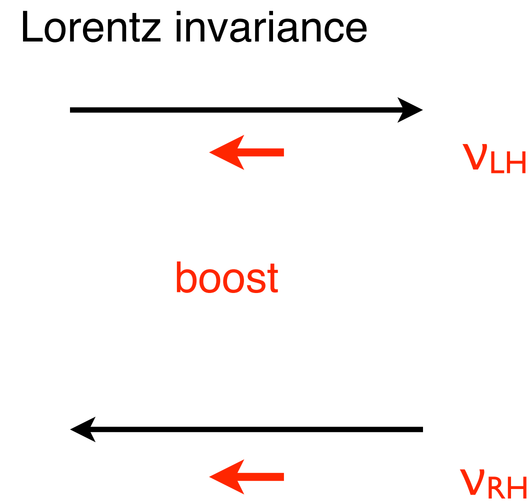
neutrino mass restores $\beta\beta$ decay as a definitive test of lepton number violation, though with a rate suppressed by $(m_\nu/E_\nu)^2$ where $E_\nu \sim 1/R_{nuclear}$

That is, if the mass is a *Majorana mass*, both selection rules forbidding this process are defeated. But if Dirac, one remains.

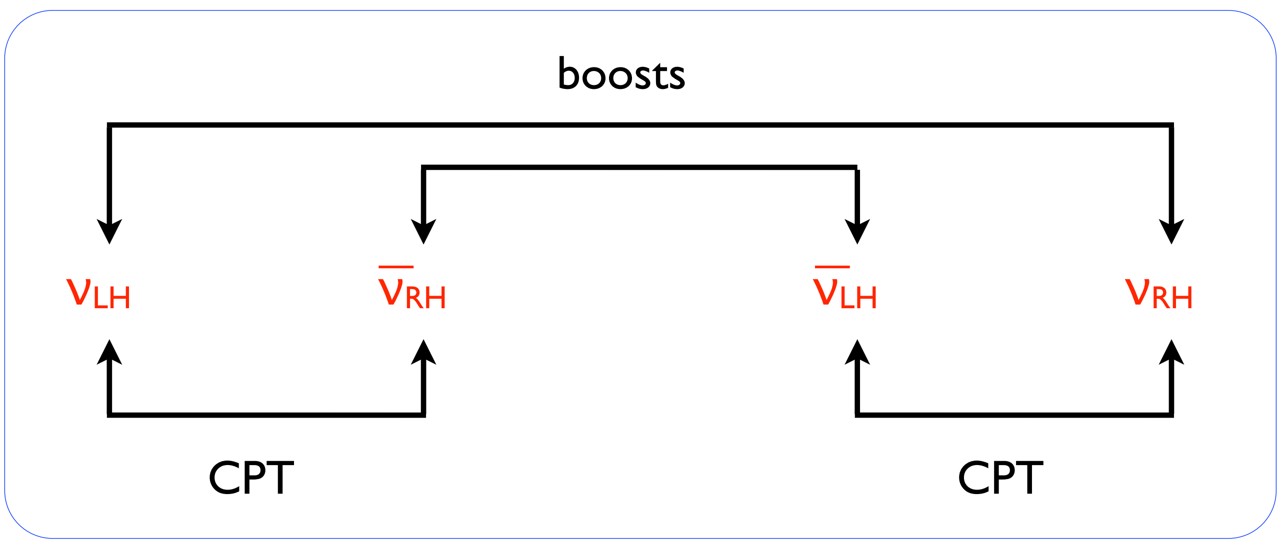
We have been discussing two limits for describing massive neutrinos

Majorana:





Dirac:



Let's see the mass consequences: start with the Dirac eq., project out

$$\psi_{R/L} = \frac{1}{2}(1 \pm \gamma_5)\psi] \qquad C\ \psi_{R/L}\ C^{-1} = \psi^c_{R/L}$$

Allow for multiple flavors and flavor mixing

$$L_m(x) \sim m_D \bar{\psi}(x)\psi(x) \Rightarrow M_D \bar{\Psi}(x)\Psi(x) \qquad \Psi_L \equiv \begin{pmatrix} \Psi^e_L \\ \Psi^\mu_L \\ \Psi^\tau_L \end{pmatrix}$$

Gives a 4n by 4n matrix, n the number of generations

$$(\bar{\Psi}^c_L, \bar{\Psi}_R, \bar{\Psi}_L, \bar{\Psi}^c_R) \begin{pmatrix} 0 & 0 & & M_D^T \\ 0 & 0 & M_D & \\ & M_D^\dagger & 0 & 0 \\ M_D^* & & 0 & 0 \end{pmatrix} \begin{pmatrix} \Psi^c_L \\ \Psi_R \\ \Psi_L \\ \Psi^c_R \end{pmatrix}$$

The Majorana mass terms complete this matrix

$$L_M = \left[\bar{\Psi}_R M_D \Psi_L + \bar{\Psi}^c_L M_D^T \Psi^c_R + \bar{\Psi}^c_L M_L \Psi_L + \bar{\Psi}^c_R M_R \Psi_R\right] + h.c.$$

$$= \left(\bar{\Psi}^c_L, \bar{\Psi}_R, \bar{\Psi}_L, \bar{\Psi}^c_R\right) \begin{pmatrix} 0 & 0 & M_L & M_D^T \\ 0 & 0 & M_D & M_R^\dagger \\ M_L^\dagger & M_D^\dagger & 0 & 0 \\ M_D^* & M_R & 0 & 0 \end{pmatrix} \begin{pmatrix} \Psi^c_L \\ \Psi_R \\ \Psi_L \\ \Psi^c_R \end{pmatrix}$$

The Majorana mass terms complete this matrix

$$L_M = \left[\bar{\Psi}_R M_D \Psi_L + \bar{\Psi}^c_L M_D^T \Psi^c_R + \bar{\Psi}^c_L M_L \Psi_L + \bar{\Psi}^c_R M_R \Psi_R\right] + h.c.$$

$$= \left(\bar{\Psi}^c_L, \bar{\Psi}_R, \bar{\Psi}_L, \bar{\Psi}^c_R\right) \begin{pmatrix} 0 & 0 & M_L & M_D^T \\ 0 & 0 & M_D & M_R^\dagger \\ M_L^\dagger & M_D^\dagger & 0 & 0 \\ M_D^* & M_R & 0 & 0 \end{pmatrix} \begin{pmatrix} \Psi^c_L \\ \Psi_R \\ \Psi_L \\ \Psi^c_R \end{pmatrix}$$

The SM: 1) has no RHed ν fields ⇒ no Dirac masses

2) renormalizable ⇒ no Majorana masses

so massless SM neutrinos

The Majorana mass terms complete this matrix

$$L_M = \left[\bar{\Psi}_R M_D \Psi_L + \bar{\Psi}_L^c M_D^T \Psi_R^c + \bar{\Psi}_L^c M_L \Psi_L + \bar{\Psi}_R^c M_R \Psi_R\right] + h.c.$$

$$= \left(\bar{\Psi}_L^c, \bar{\Psi}_R, \bar{\Psi}_L, \bar{\Psi}_R^c\right) \begin{pmatrix} 0 & 0 & M_L & M_D^T \\ 0 & 0 & M_D & M_R^\dagger \\ M_L^\dagger & M_D^\dagger & 0 & 0 \\ M_D^* & M_R & 0 & 0 \end{pmatrix} \begin{pmatrix} \Psi_L^c \\ \Psi_R \\ \Psi_L \\ \Psi_R^c \end{pmatrix}$$

But 1) might anticipate M_D ~ other SM Dirac masses

2) know $M_L \ll M_D$ (no ββ decay), reasonably $M_R \gg M_D$

so with these assumptions can diagonalize this matrix

The Majorana mass terms complete this matrix

$$L_M = \left[\bar{\Psi}_R M_D \Psi_L + \bar{\Psi}^c_L M_D^T \Psi^c_R + \bar{\Psi}^c_L M_L \Psi_L + \bar{\Psi}^c_R M_R \Psi_R\right] + h.c.$$

$$= (\bar{\Psi}^c_L, \bar{\Psi}_R, \bar{\Psi}_L, \bar{\Psi}^c_R) \begin{pmatrix} 0 & 0 & M_L & M_D^T \\ 0 & 0 & M_D & M_R^\dagger \\ M_L^\dagger & M_D^\dagger & 0 & 0 \\ M_D^* & M_R & 0 & 0 \end{pmatrix} \begin{pmatrix} \Psi^c_L \\ \Psi_R \\ \Psi_L \\ \Psi^c_R \end{pmatrix}$$

$$m_\nu^{\text{light}} = M_D \left(\frac{M_D}{M_R}\right)$$

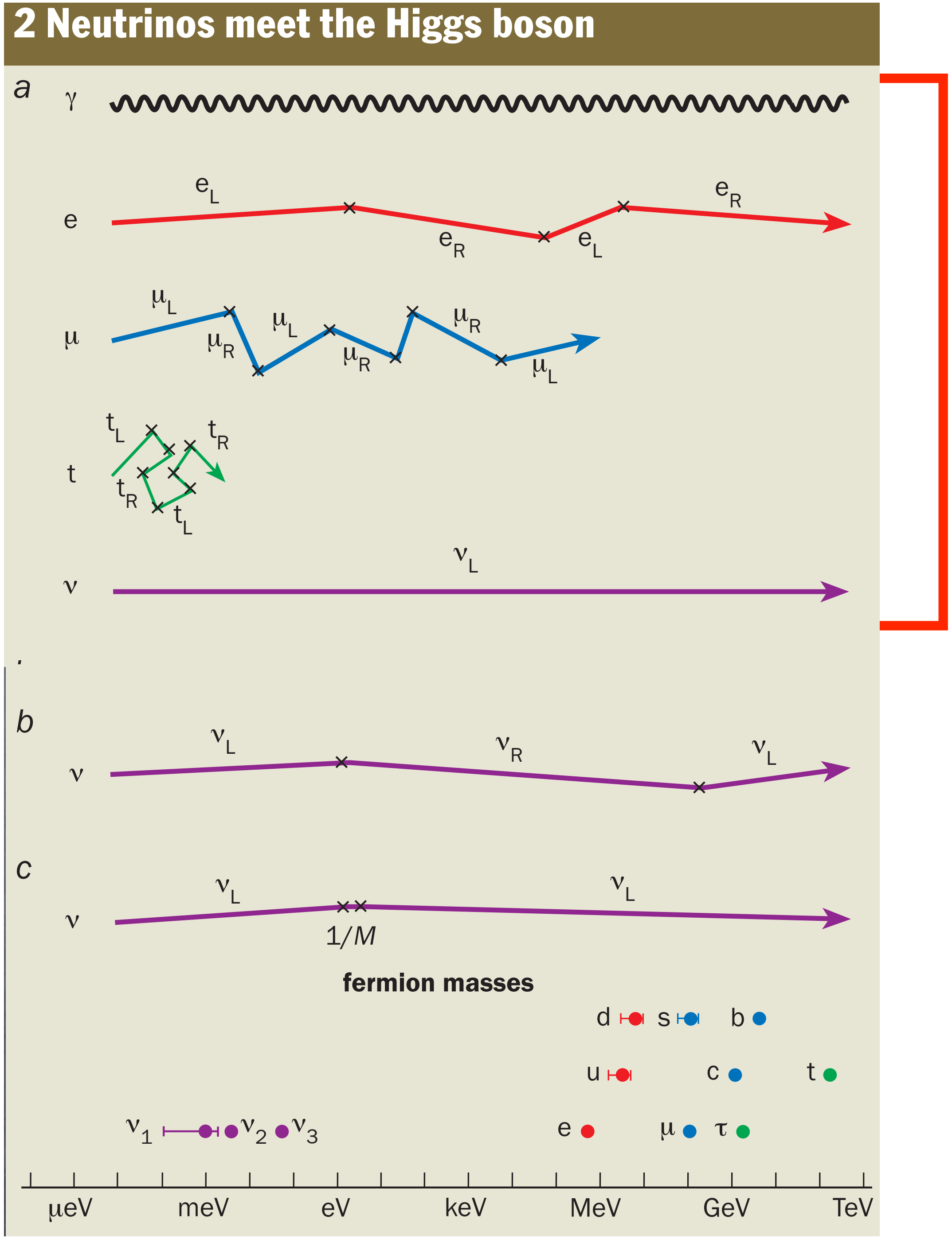
seesaw

SM fermion mass scale

needed “small parameter” specific to vs

Gell-Mann, Ramond, and Slansky 80; Yanagida

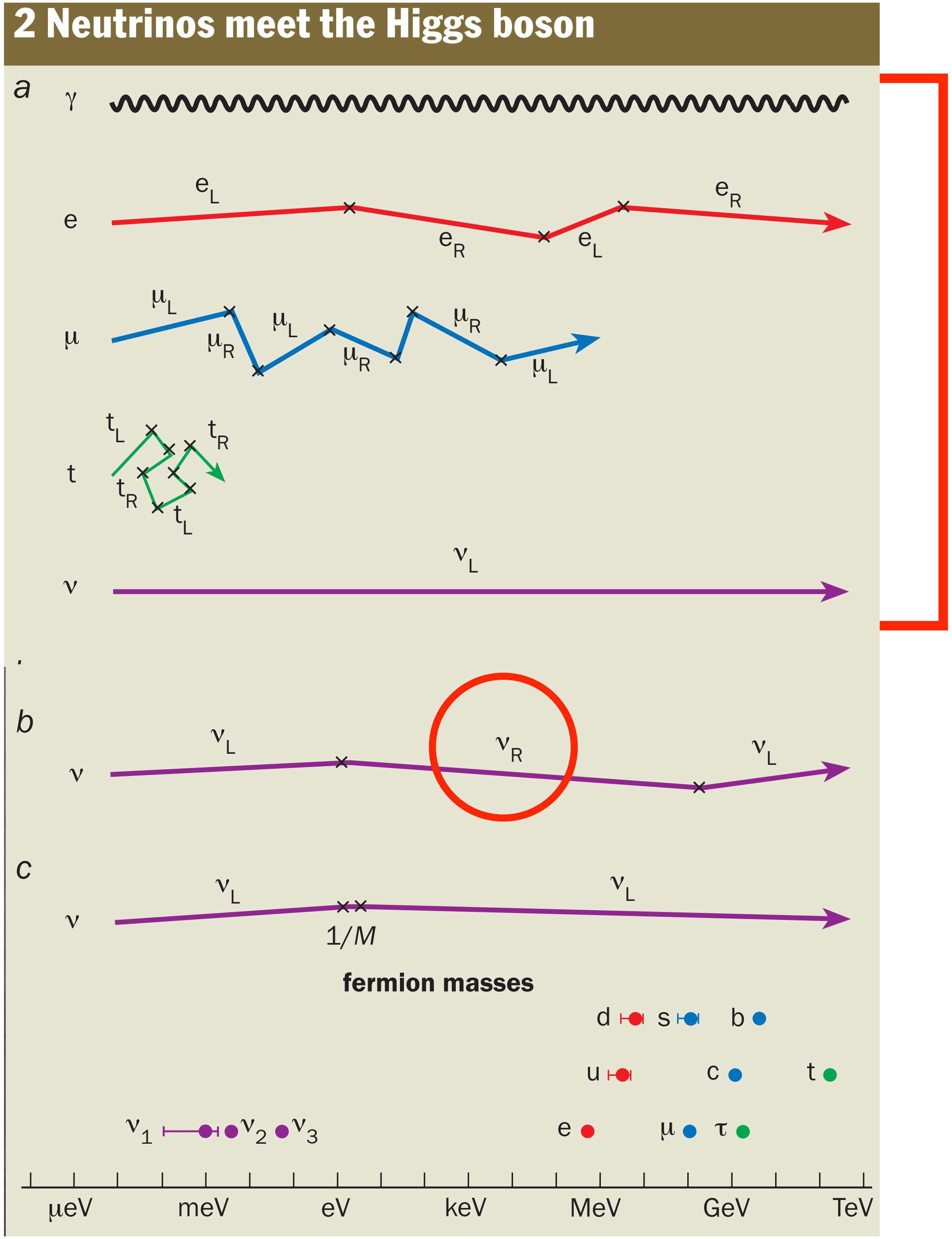
Murayama's ν mass cartoon



standard model fermion masses

standard model ν and mass=0

Murayama's ν mass cartoon

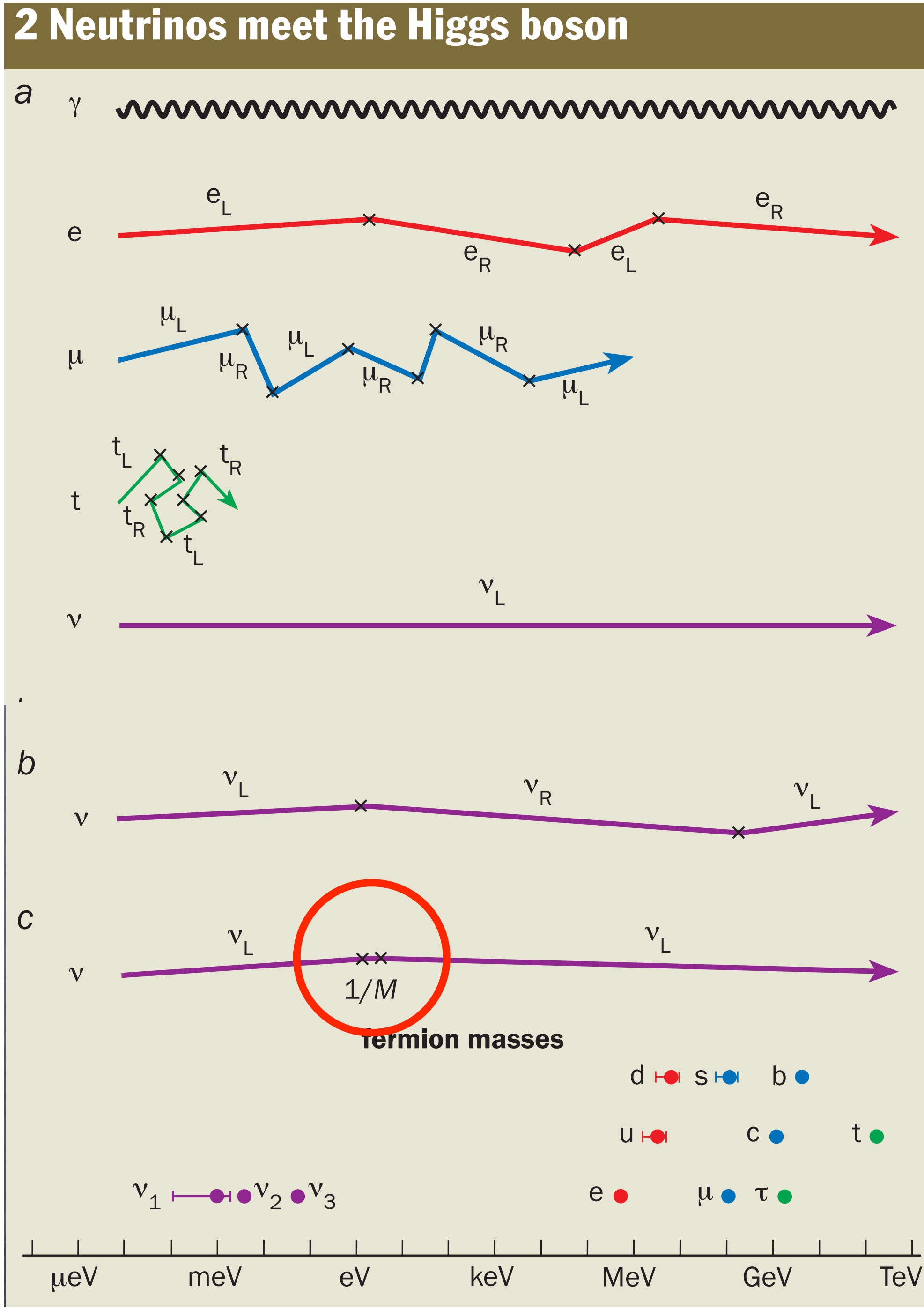


standard model fermion masses

standard model ν and mass=0

light Dirac neutrino mass

Murayama's ν mass cartoon



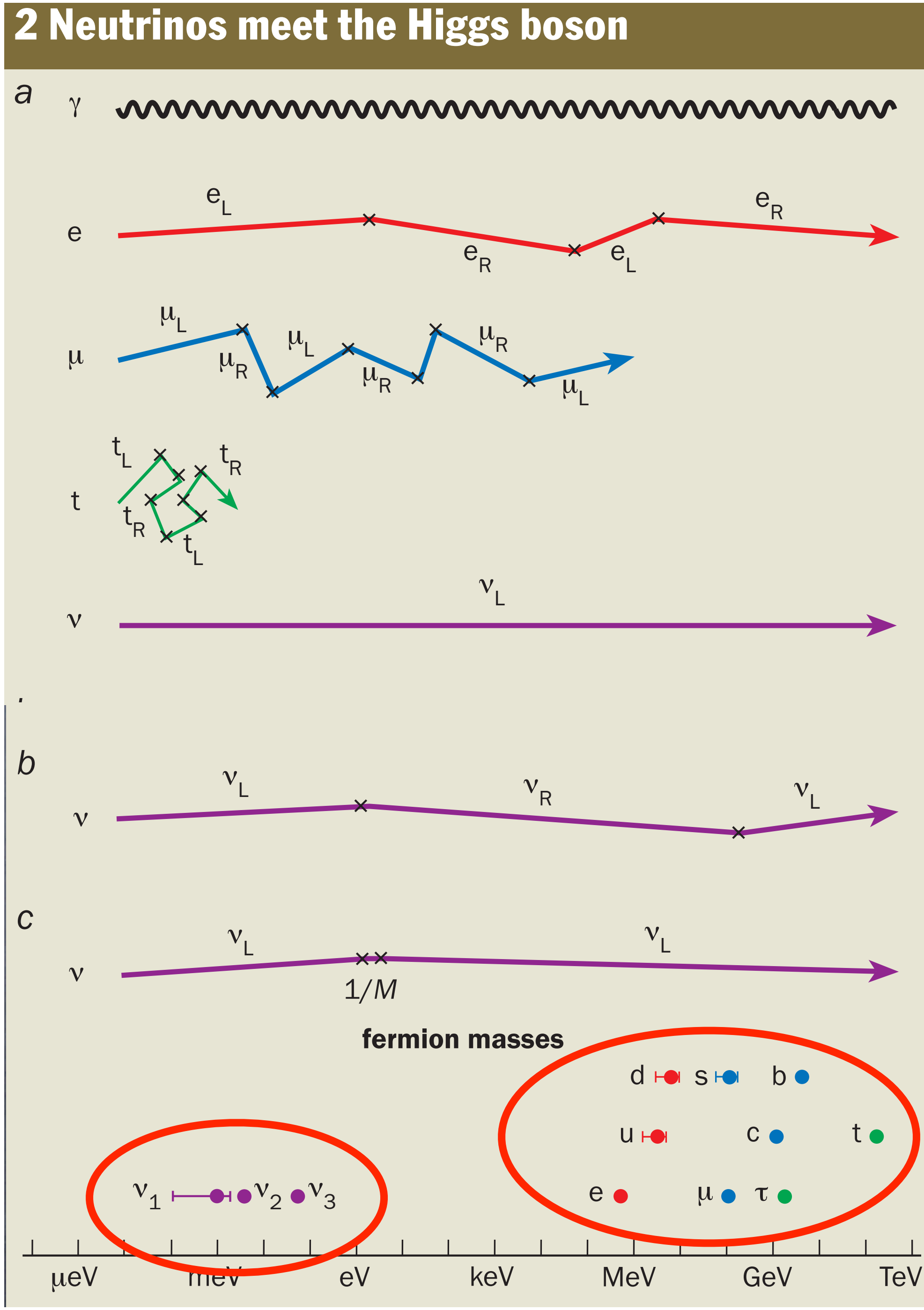
standard model fermion masses

standard model ν and mass=0

light Dirac neutrino mass

light LHed Majorana neutrino mass

Murayama's ν mass cartoon



standard model fermion masses

standard model ν and mass=0

light Dirac neutrino mass

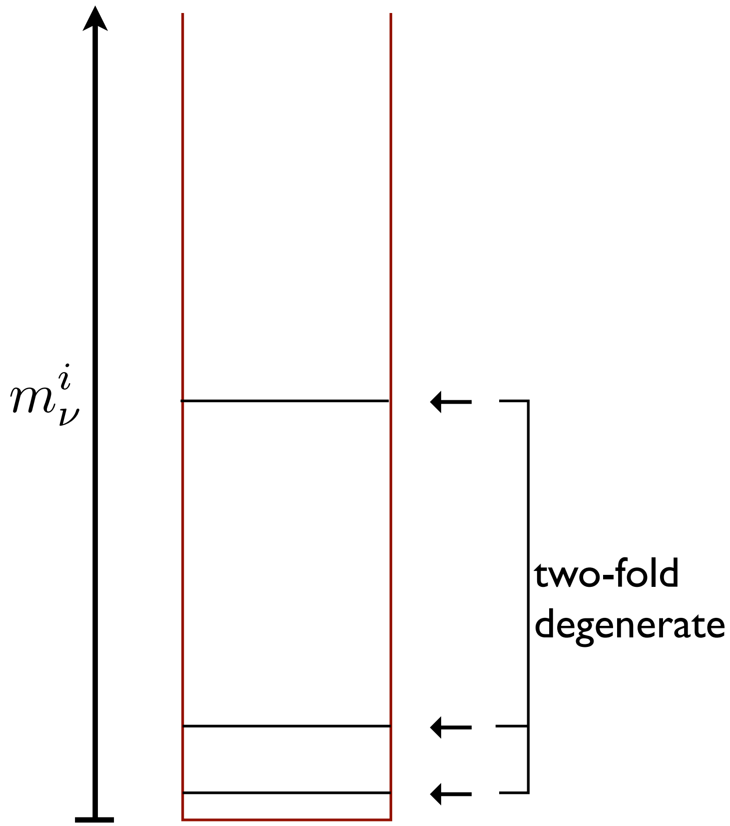
light LHed Majorana neutrino mass

← the anomalous ν mass scale, connected with the seesaw?

- The ν mass tested in $\beta\beta$ decay is

$$\langle m_{\beta\beta}\rangle = \sum_{i=1}^{2n} U^L_{\rm ei} U^L_{\rm ei}\ \lambda_i^{\rm CP}\ m_\nu^i$$

some scenarios



Dirac limit:

$$M_L,\ M_R \equiv 0$$

mass degeneracy ⇒

patch two eigenstates together ⇒

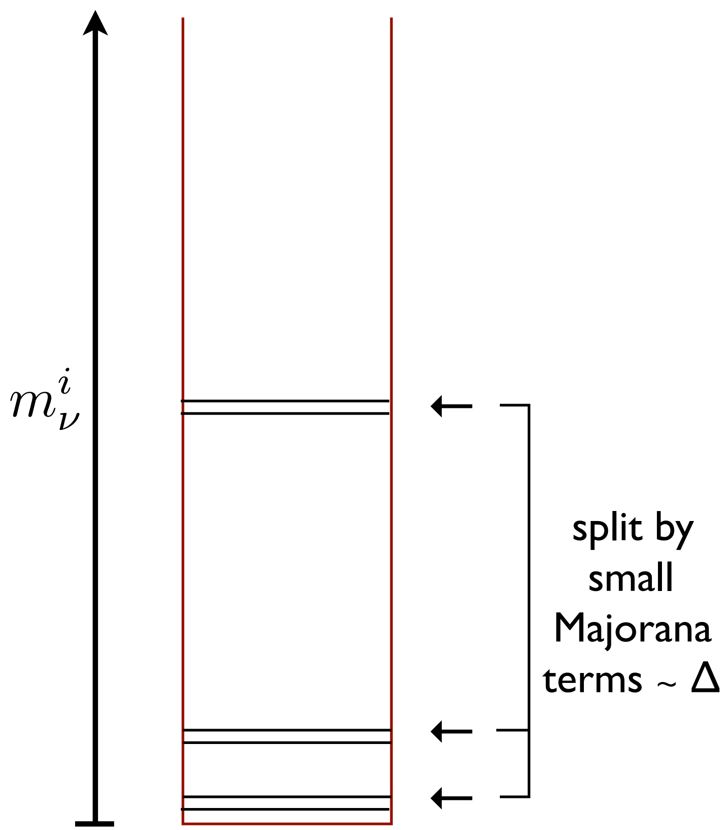
2 × 2 component Dirac spinors

$$\langle m_{\beta\beta}\rangle \equiv 0$$

- The ν mass tested in $\beta\beta$ decay is

$$\langle m_{\beta\beta}\rangle = \sum_{i=1}^{2n} U^L_{\mathrm{ei}} U^L_{\mathrm{ei}}\ \lambda_i^{\mathrm{CP}}\ m_\nu^i$$

some scenarios



Pseudo-Dirac limit:

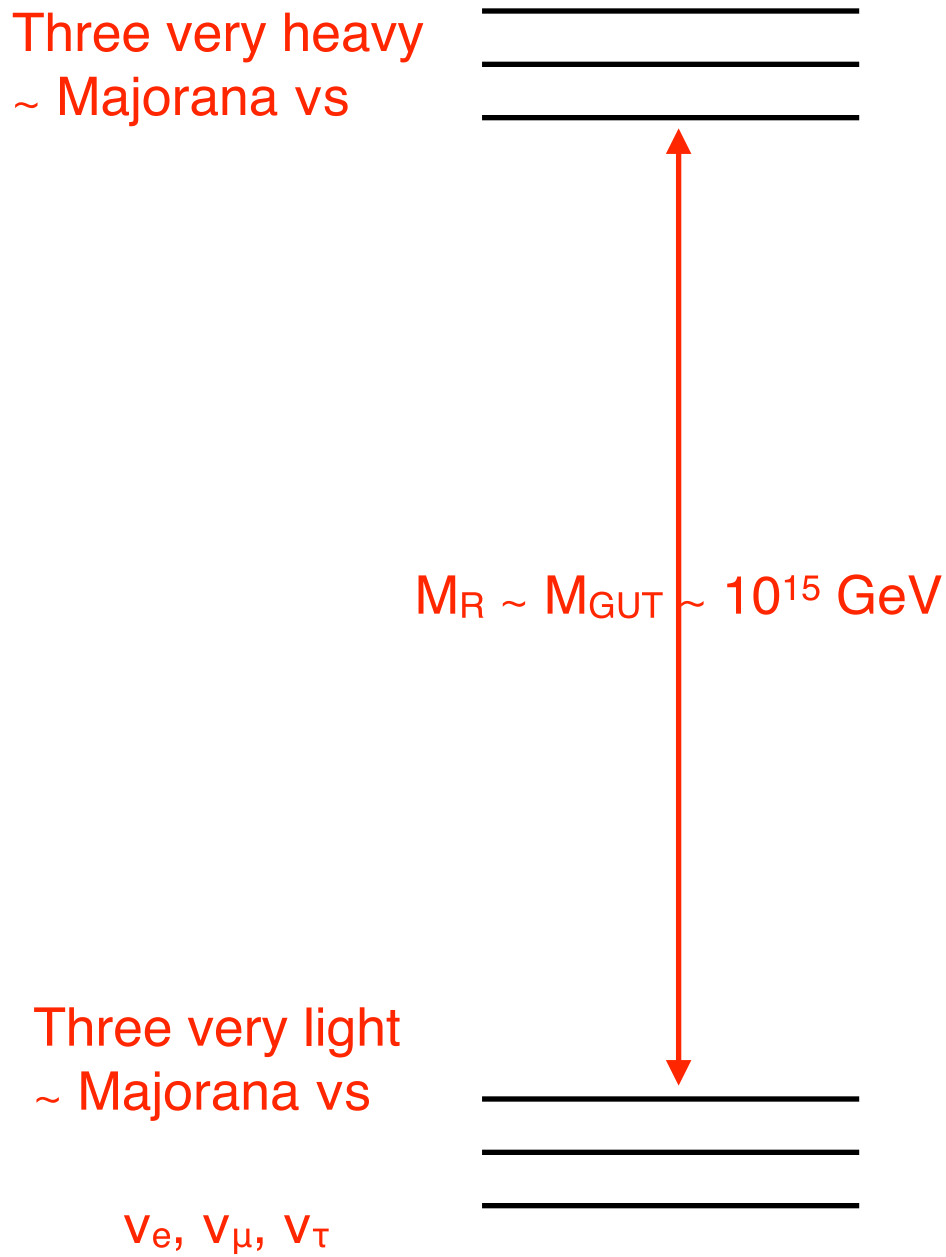
$$M_L, M_R << M_D$$

mass doublets ⇒

could appear experimentally as just three states

$$\langle m_{\beta\beta}\rangle \sim \Delta$$

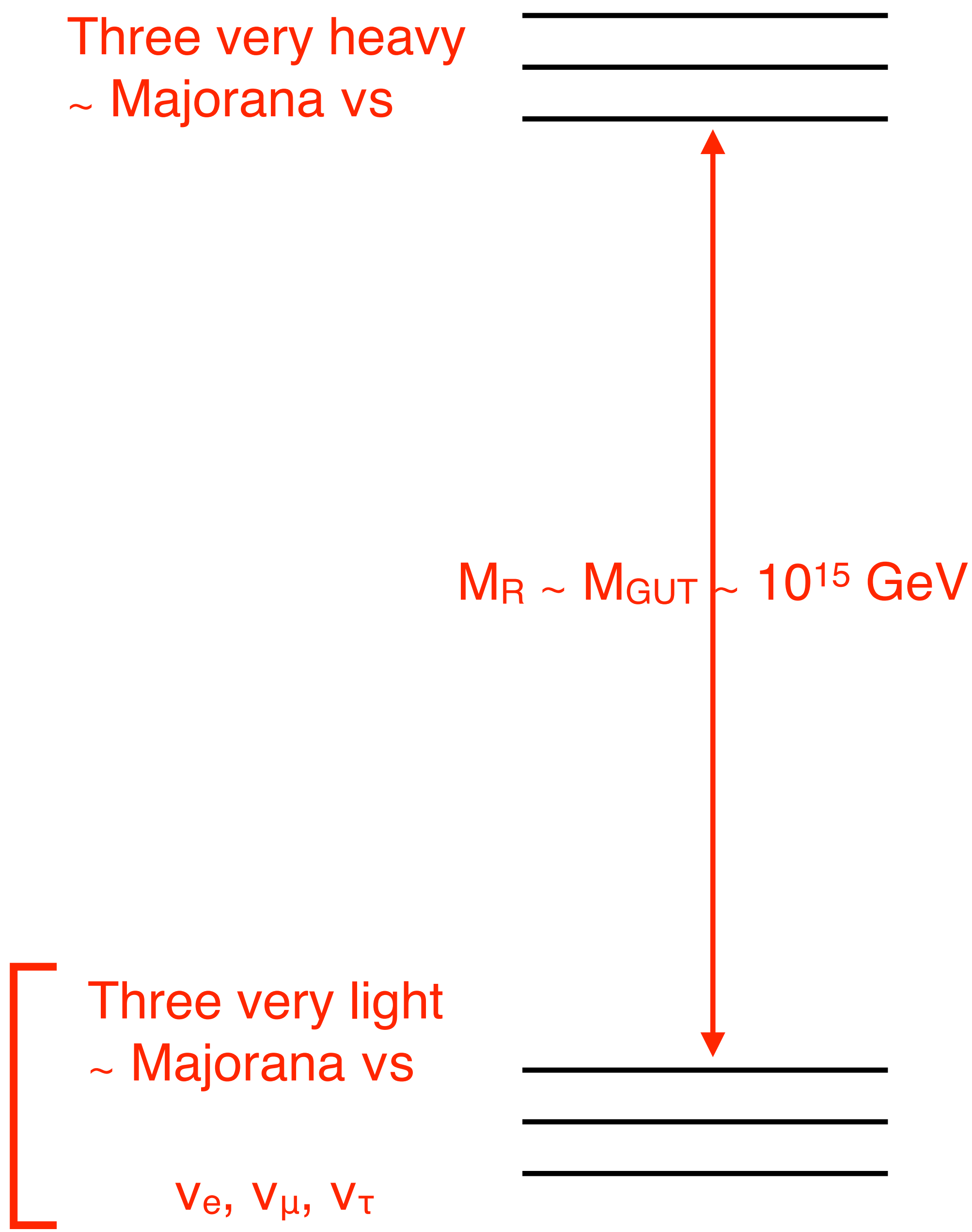
Has led to a “standard scenario” that is used to discuss ββ decay and other experiments



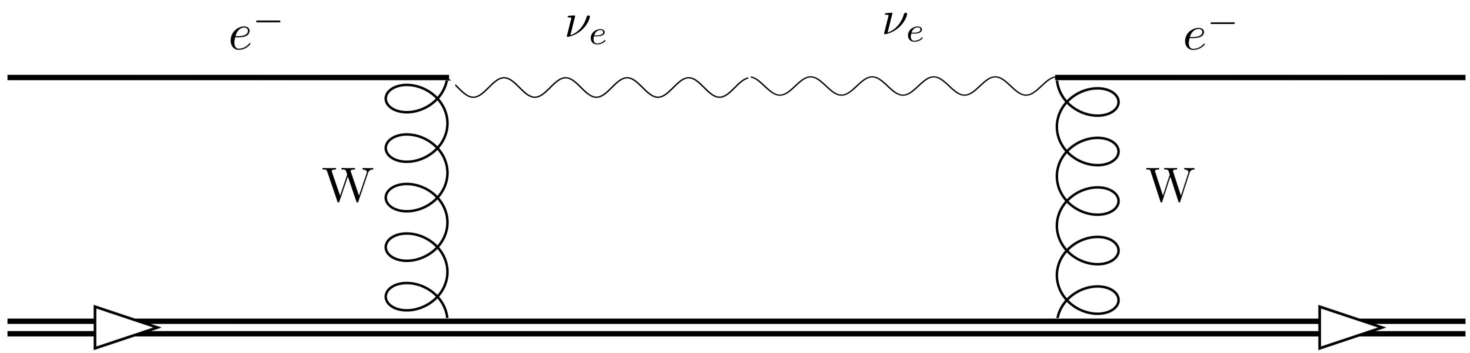
Alternatively, $M_R >> M_D$

This is the “standard scenario” that is used to discuss ββ decay and other experiments

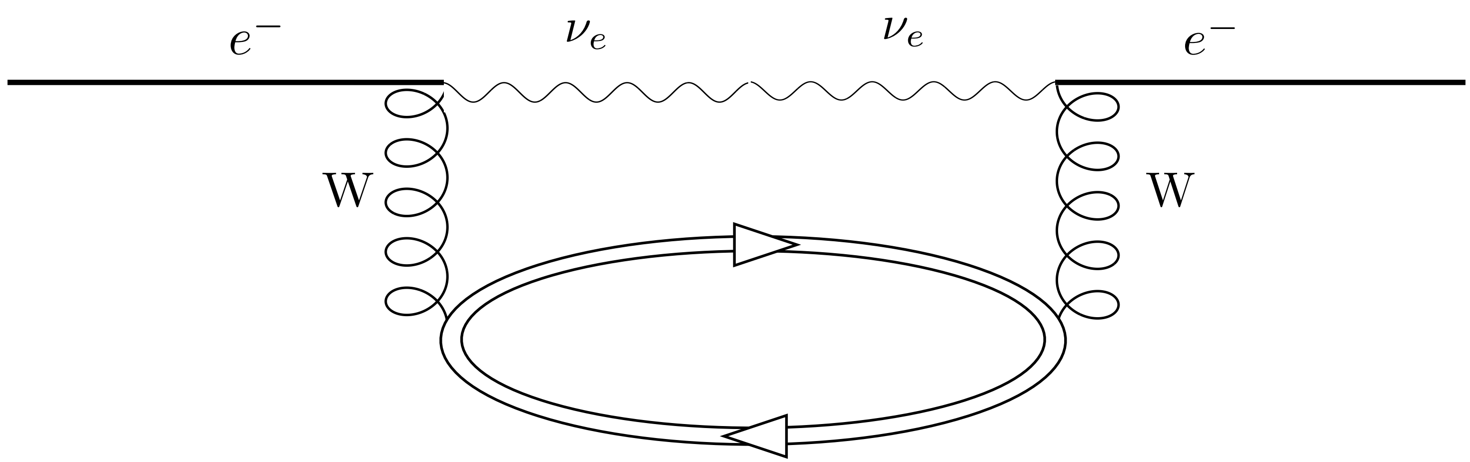
properties being probed in low energy experiments, cosmology



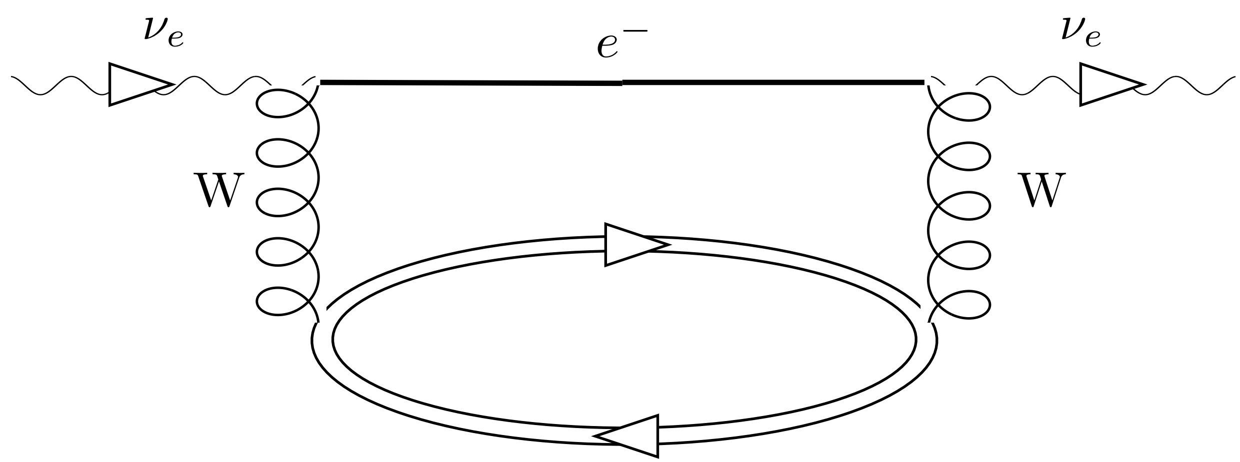
The observation of neutrinoless ββ decay ~ implies neutrino mass



The observation of neutrinoless ββ decay ~ implies neutrino mass

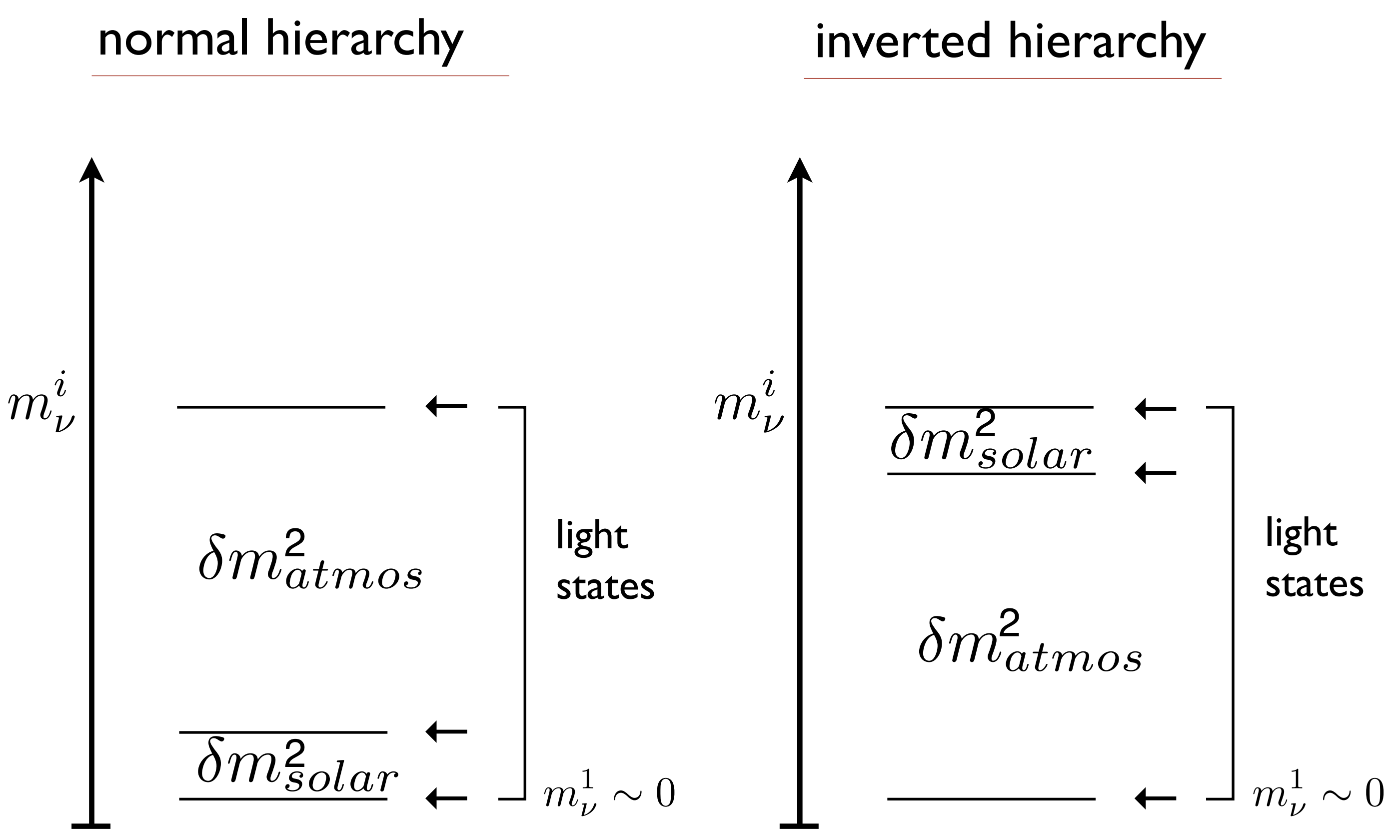


The observation of neutrinoless ββ decay ~ implies neutrino mass



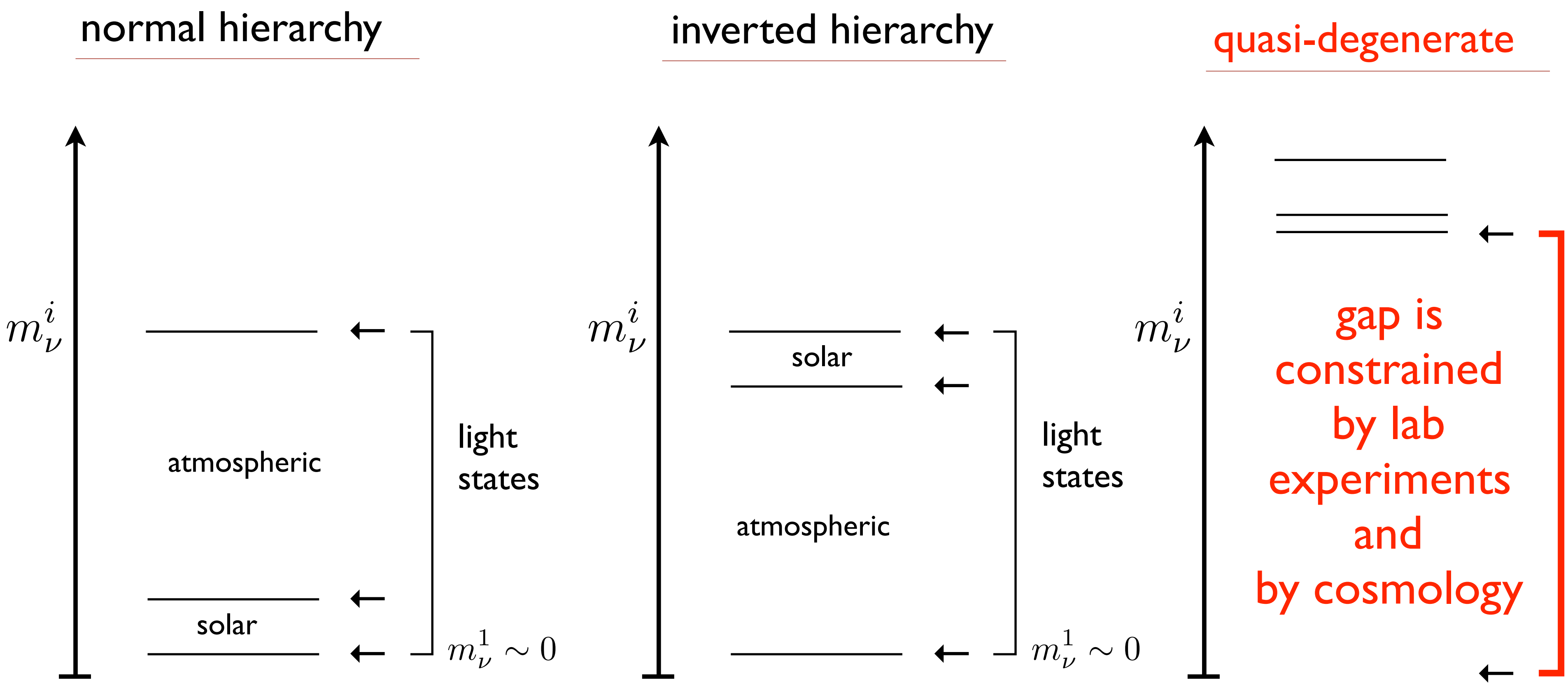
(Valle & Schecter “black box” ν mass mechanism)

We have learned a lot about about the pattern of the light masses from the solar, atmospheric, reactor, and accelerator experiments - but two hierarchies remain

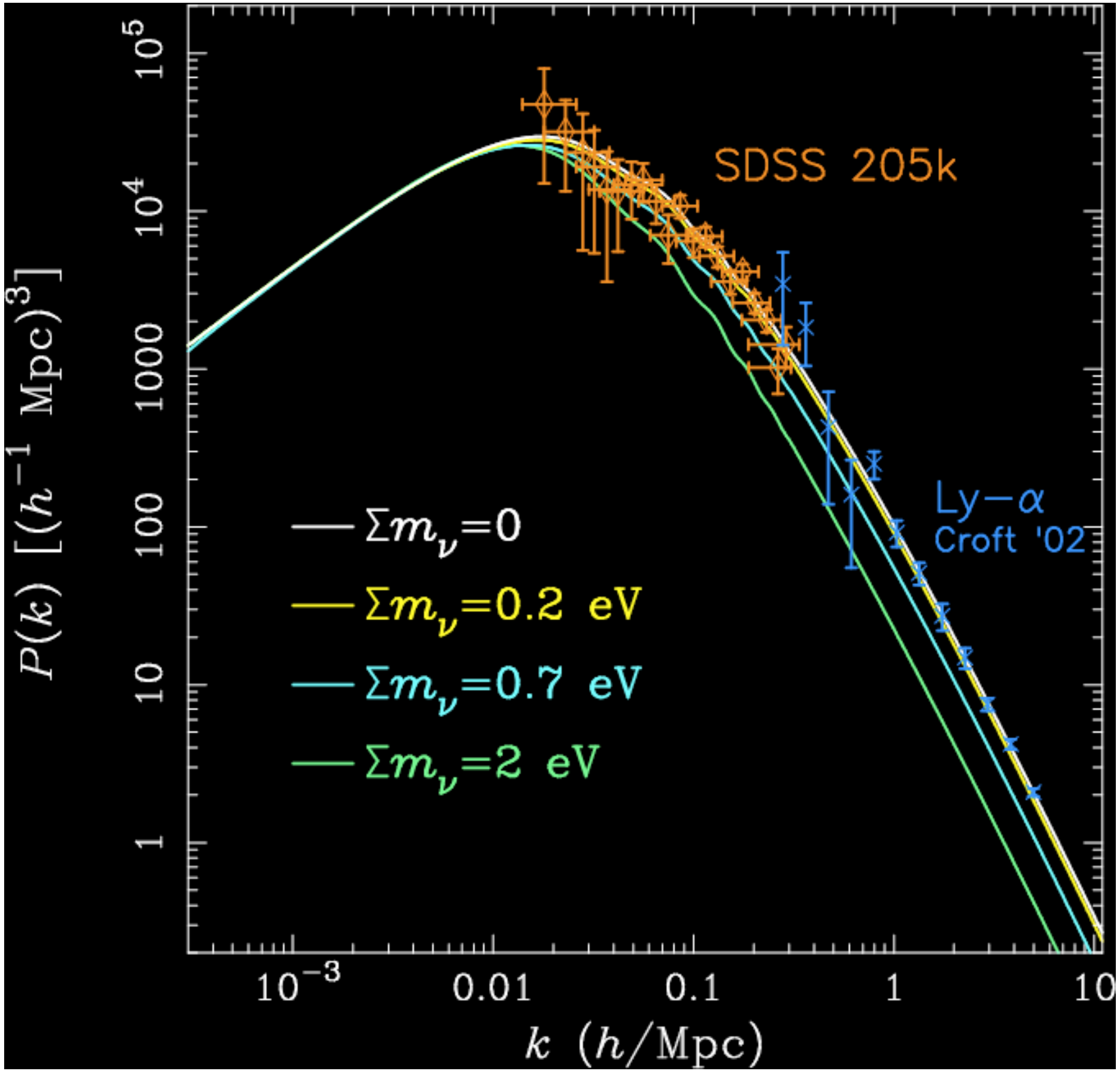


We also do not know the absolute scale of the masses

Oscillations measure mass differences $\delta m^2_{21} = \delta m^2_{solar}, \delta m^2_{31} = \delta m^2_{atmos}$
The absolute scale is not fixed



how do we measure absolute masses?



Cosmology limits the quasi-degenerate case

ν's start off relativistic in the early universe, where they suppress the growth of structure on large scales

Transition to nonrelativistic

Effects scale and redshift dependent

Current limits (Planck2015)

$$\frac{1}{3}\sum_i m_i \lesssim 80\ \mathrm{meV}$$

We have learned a lot about mixing angles:

$$\text{undetermined}: \delta; \quad \phi_1, \phi_2$$

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} c_{12}c & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \begin{pmatrix} \nu_1 \\ e^{i\phi_1}\nu_2 \\ e^{i\phi_2}\nu_3 \end{pmatrix}$$

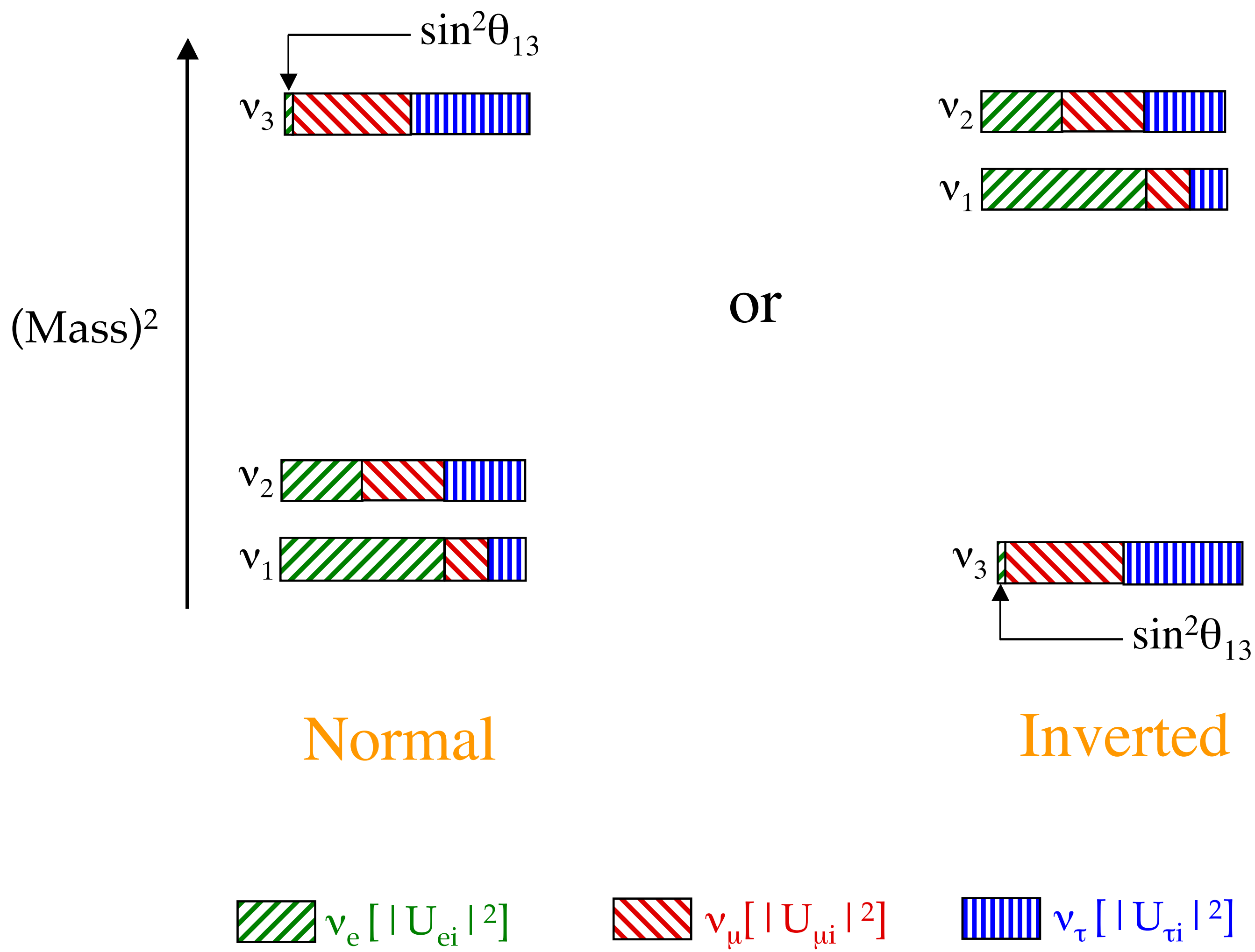
$$= \begin{pmatrix} 1 & & \\ & c_{23} & s_{23} \\ & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & & s_{13}e^{-i\delta} \\ & 1 & \\ -s_{13}e^{i\delta} & & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & \\ -s_{12} & c_{12} & \\ & & 1 \end{pmatrix} \begin{pmatrix} \nu_1 \\ e^{i\phi_1}\nu_2 \\ e^{i\phi_2}\nu_3 \end{pmatrix}$$

Atmospheric | Reactor (Daya Bay/Reno/Double Chooz) | Solar

$$\sin^2\theta_{23} = \begin{cases} 0.386^{+0.024}_{-0.021}, & \text{NH} \\ 0.392^{+0.039}_{-0.022} & \text{IH} \end{cases} \qquad \sin^2\theta_{13} = \begin{cases} 0.0241 \pm 0.0025, & \text{NH} \\ 0.0244^{+0.0023}_{-0.0025}, & \text{IH} \end{cases} \qquad \sin^2\theta_{12} = 0.307^{+0.018}_{-0.016}$$

Bari global analysis
(Valencia quite similar)

Or graphically:



$$\delta m^2_{31} = \begin{cases} (2.47^{0.06}_{-0.10}) \times 10^{-3}\text{eV}^2, & \text{NH} \\ -(2.37^{+0.07}_{0.11}) \times 10^{-3}\text{eV}^2, & \text{IH} \end{cases}$$

$$\delta m^2_{21} = 7.54^{+0.26}_{-0.22} \times 10^{-5}\text{eV}^2$$

Bari global analysis
(Valencia quite similar)

Δ_{12} $\quad|\Delta_{23}|$ $\quad\text{sign}[\Delta_{23}]$ $\quad$ absolute scale

Plugging in the measured ν mass matrix parameters

NH:

$$\langle m_{\beta\beta}\rangle \sim \left|\sqrt{\delta m_{21}^2}\ s_{12}^2 c_{13}^2 + \sqrt{|\delta m_{31}^2|}\ s_{13}^2 e^{i\phi}\right| \sim |4.8 + 1.2 e^{i\phi}|\ \mathrm{meV}$$

IH:

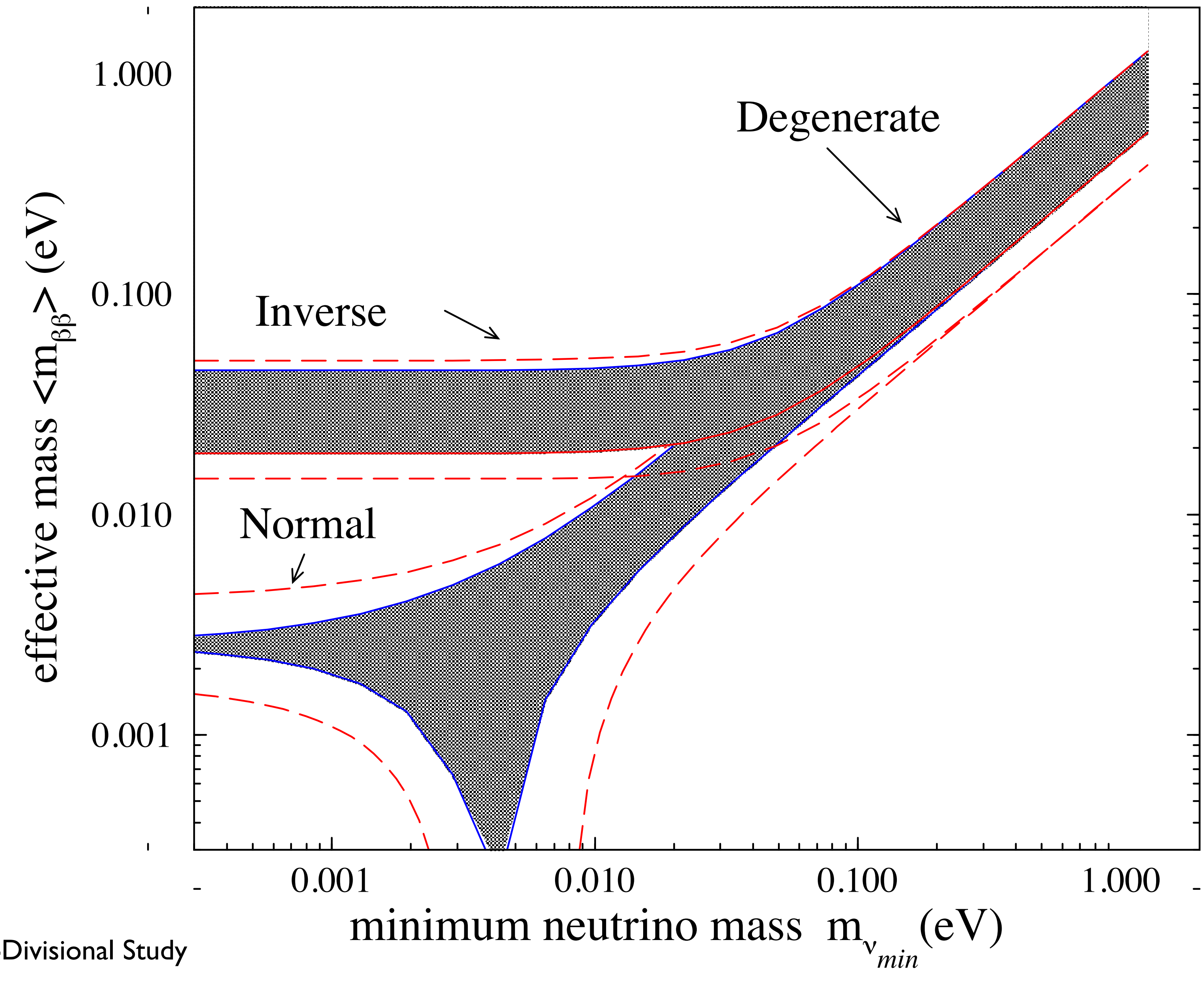
$$\langle m_{\beta\beta}\rangle \sim \sqrt{|\delta m_{31}^2|}\ c_{13}^2\ \sqrt{1 - \sin^2 2\theta_{12} \sin^2\phi} = [19 \leftrightarrow 49]\ \mathrm{meV}$$

QD:

$$\langle m_{\beta\beta}\rangle \sim m_0 \left|c_{12}^2 c_{13}^2 e^{i\phi} + s_{12}^2 c_{13}^2 e^{i\phi'} + s_{13}^2\right| \sim m_0(0.68 \pm 0.32)$$

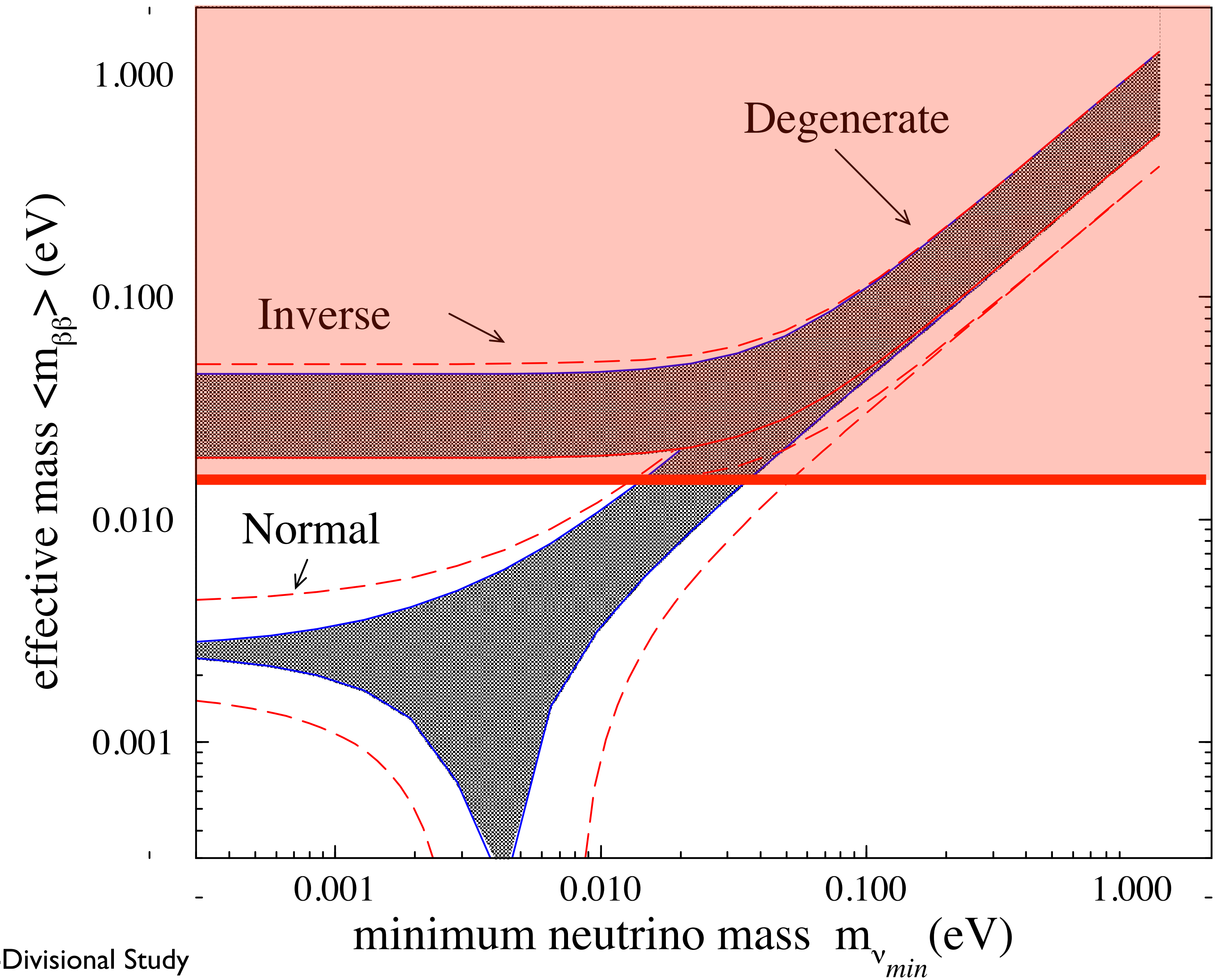
$$m_0 \lesssim 80\ \mathrm{meV}\quad \mathrm{Planck,\ BAO,\ ....}$$

Mass scenarios critical to next-generate ββ decay efforts



APS Multi-Divisional Study

Mass scenarios critical to next-generate ββ decay efforts

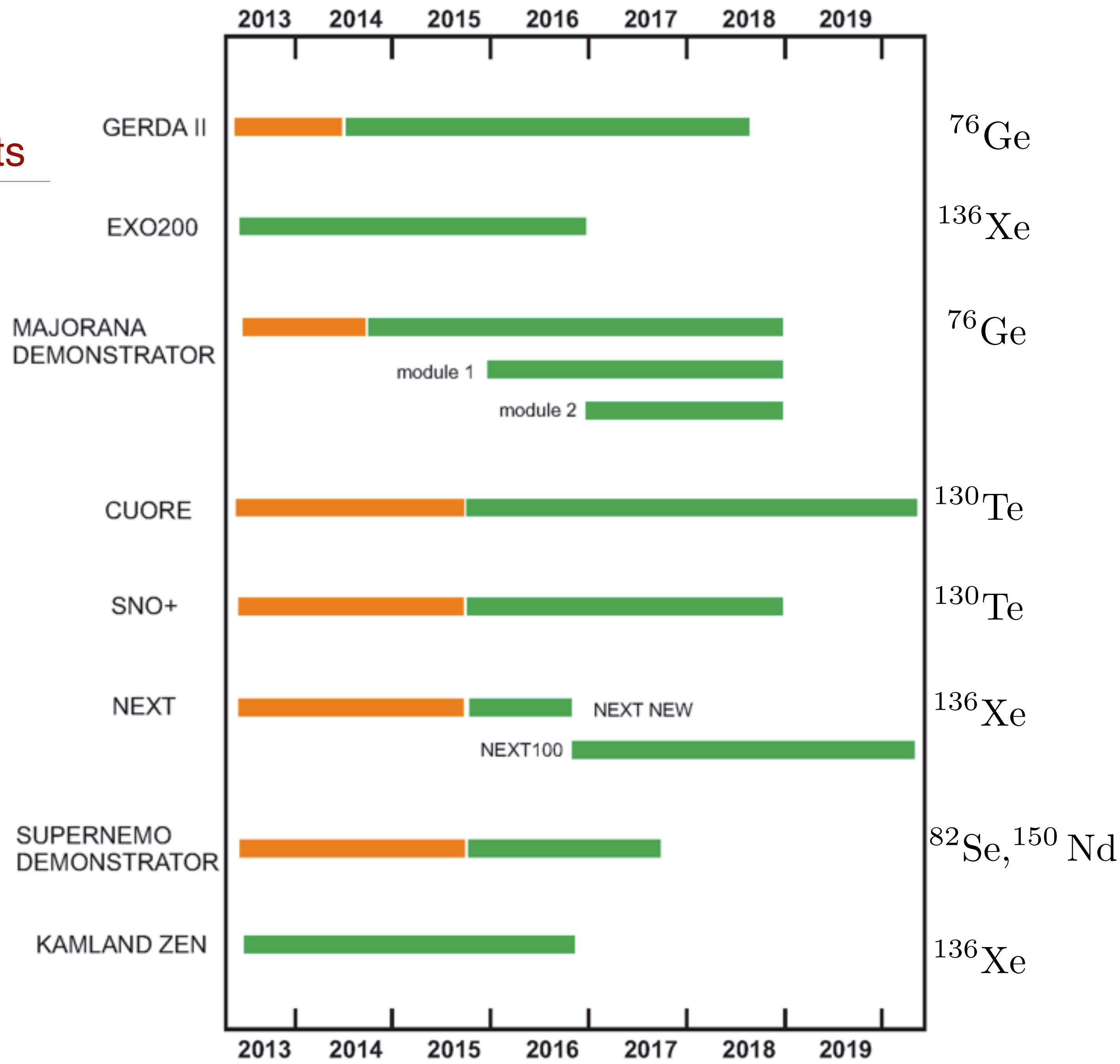


test ββ decay at the inverted hierarchy level

APS Multi-Divisional Study

‘Demonstrator’ Experiments

Current-generation timelines for project construction and running

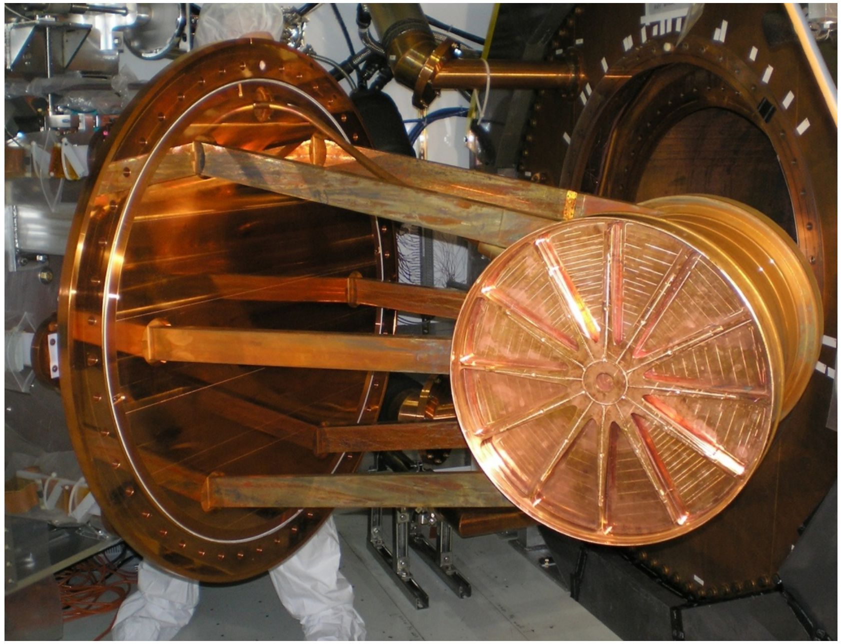


From NSAC Subcommittee on $0\nu\ \beta\beta$ decay

GERDA I, Gran Sasso; Majorana, SL

GERDA I ^{76}Ge, 21.6kg-y

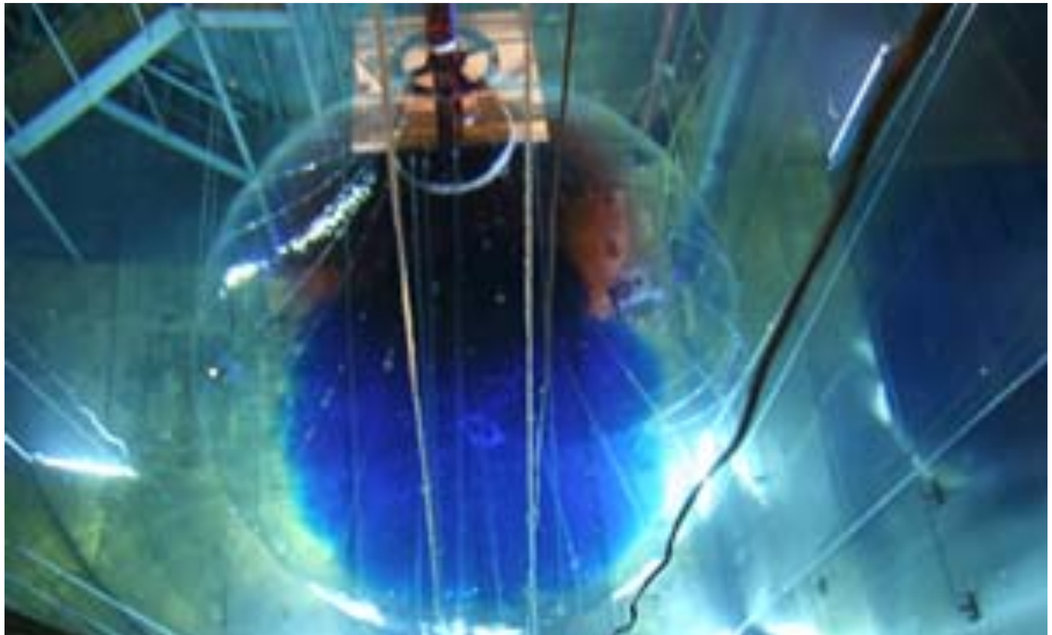
$\tau_{1/2} > 2.1 \times 10^{25}$y 90% c.l.



EXO-200, WIPP

^{136}Xe, 99.8 kg-y

$\tau_{1/2} > 1.1 \times 10^{25}$y 90% c.l.



KamLAND-Zen, Kamioka

^{136}Xe, 89.5 kg-y

$\tau_{1/2} > 1.9 \times 10^{25}$y 90% c.l.

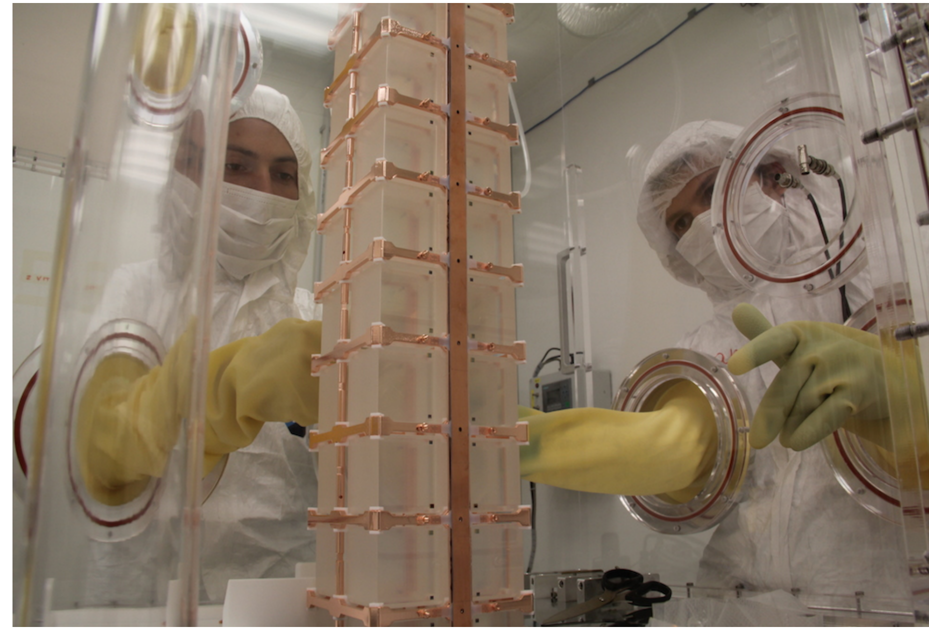
CUORE-0/Cuoricino, Gran Sasso

^{130}Te, 29.6kg-y

$\tau_{1/2} > 4.0 \times 10^{24}$y 90% c.l.

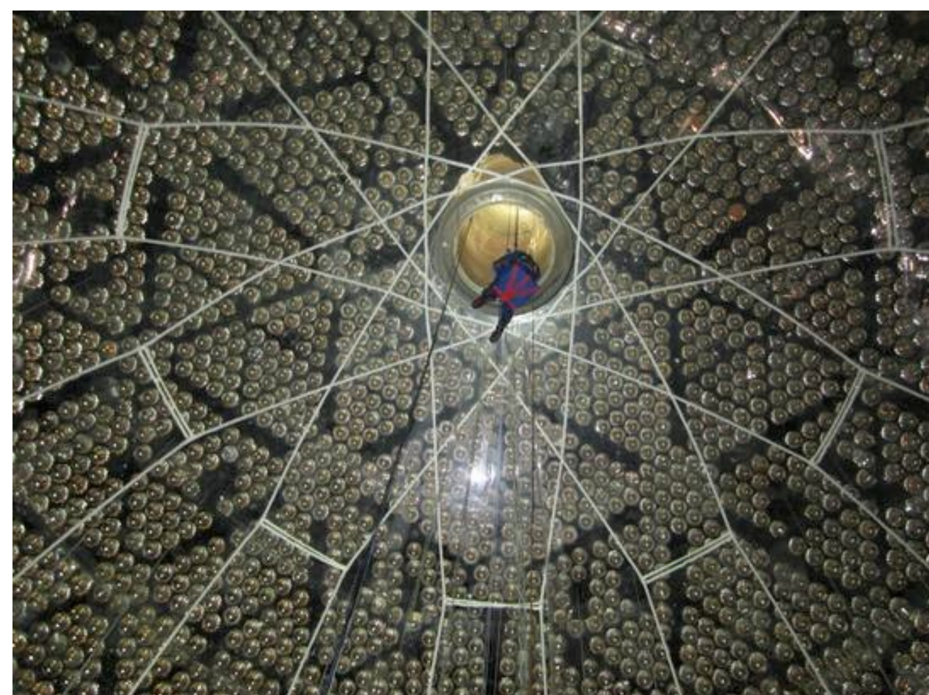
SNO+, SNOLab

^{130}Te-loaded scintillator, to begin in 2016



NEXT, Canfranc Laboratory

Gaseous ^{136}Xe TPC , final state i.d.



The benchmarks

1. where we are now

GERDA + other Ge: $\tau_{1/2} > 3.0 \times 10^{25}$y 90% c.l. $\langle m_{\beta\beta} \rangle < 460$ meV

The benchmarks

2. where the demonstrator experiments will take us

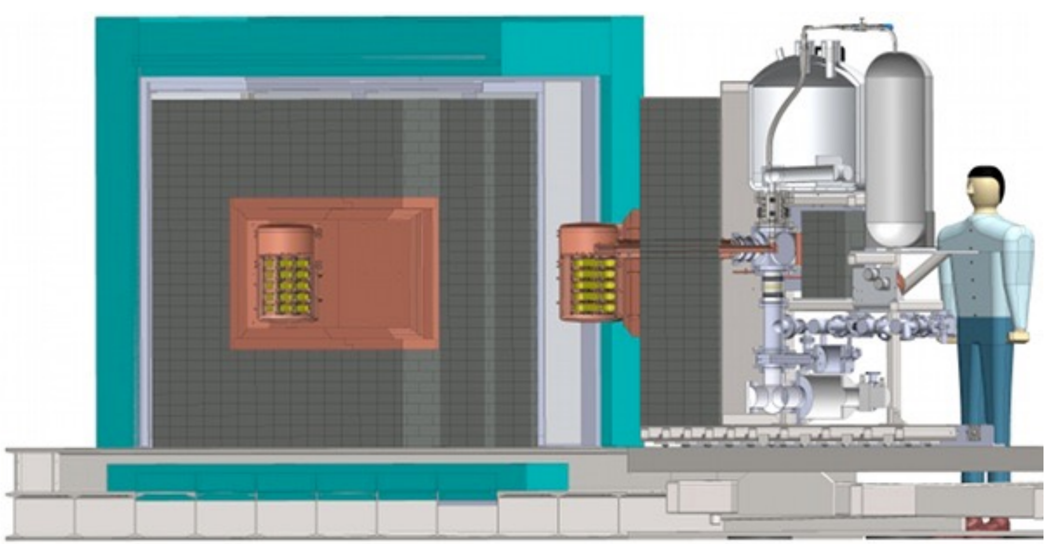
5-year ‘demonstrator’ experiments: $\sim 1.6 \times 10^{26}$ y to reach 200 meV

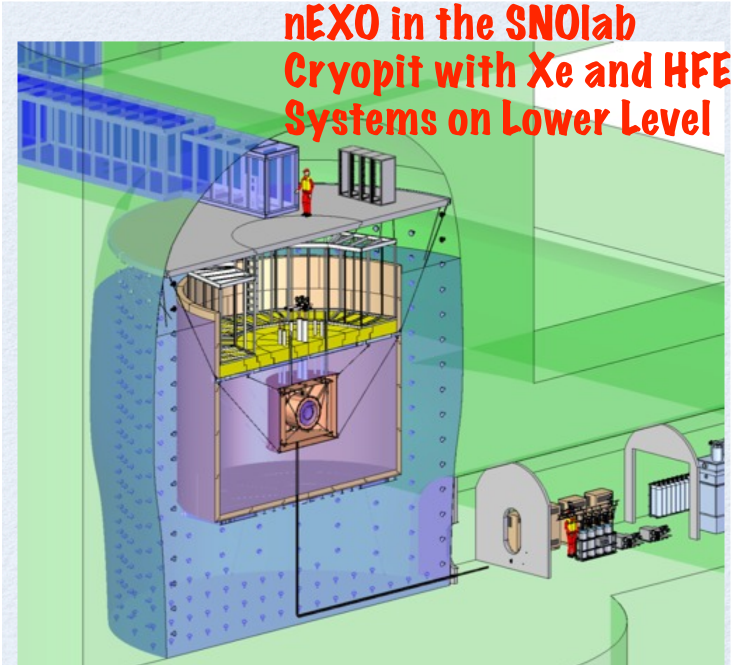
The benchmarks

3. probe the inverted hierarchy mass band of 19-49 meV

ton+ experiments reaching 10^{28} y after a decade of running

Future: One-ton Experiments 2017→ Probing the IH





Majorana and GERDA joint effort (using the best 'demonstrator' technology) a 1-ton enriched ^{76}Ge detector

EXO → nEXO at the 1-ton and then 5-ton level

desirable attributes: excellent resolution, nearly free of backgrounds, feasible costs, final-state tagging, scalability ...

Nuclear Physics: why us (theorists), why now

Rates depend on matrix elements for complex, heavy nuclei

$$M_F^{0\nu} \sim \langle 0_f^+ || \frac{1}{2} \sum_{i,j=1}^{A} \tau_+(i)\tau_+(j) F(r_{ij}) || 0_i^+ \rangle$$

$$F(r_{ij}),\ G(r_{ij}) \sim \frac{1}{r_{ij}}$$

$$M_{GT}^{0\nu} \sim \langle 0_f^+ || \frac{1}{2} \sum_{i,j=1}^{A} \vec{\sigma}(i) \cdot \vec{\sigma}(j) \tau_+(i)\tau_+(j) G(r_{ij}) || 0_i^+ \rangle$$

- the ground-state-to-ground-state transitions exhaust a tiny fraction of the underlying spin-spin sum rule ~ 0.1%

- results depend both on the quality of nuclear calculation, but also on phenomenological corrections that must be introduced to account for missing physics, such as absent high-momentum degrees of freedom, poorly understood operator renormalizations, etc

all model-based … can we do better?
this is the workshop's purpose

Some straightforward questions about NP seem to lack crisp answers

- we produce a wave function, say, in a shell model: what precisely is its relationship to the true wave function? If we cannot answer this question in a precise mathematical way, should we worry?

- our answers depend on parameter choices: how can this be?

- our wave functions are normalized to 1 and are orthogonal. As they must be low-momentum projections of the true wave functions, how can this possibly be?

- we construct an NN potential by inverting phase shifts: the detailed information in those phase shifts comes from their sharp energy dependence
    - — yet our nuclear effective interactions are energy independent
    - — isn't nature analytically continuous in E? Shouldn't reaction channels and bound state channels have the same properties? Shouldn't they be aspects of one theory?

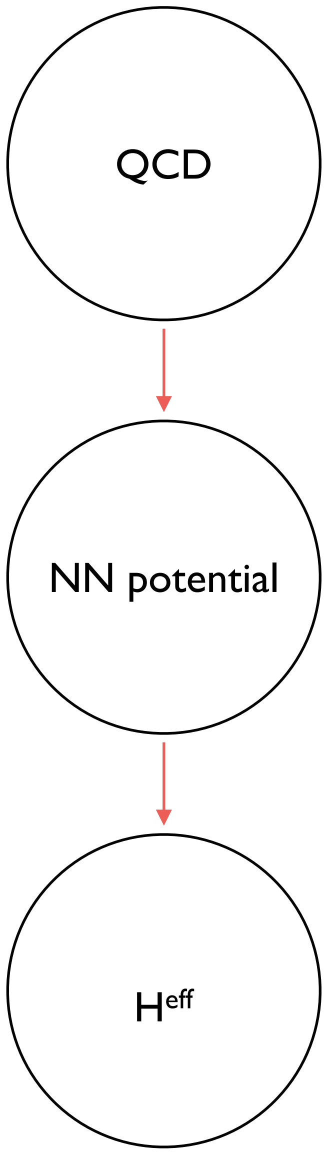
- the NN potential we so derive is awful: singular, nonperturbative
  - — impossibly difficult to renormalize rigorously for SM-like work
  - — why the two-step? what is this necessary? why not QCD -> many-body space directly??

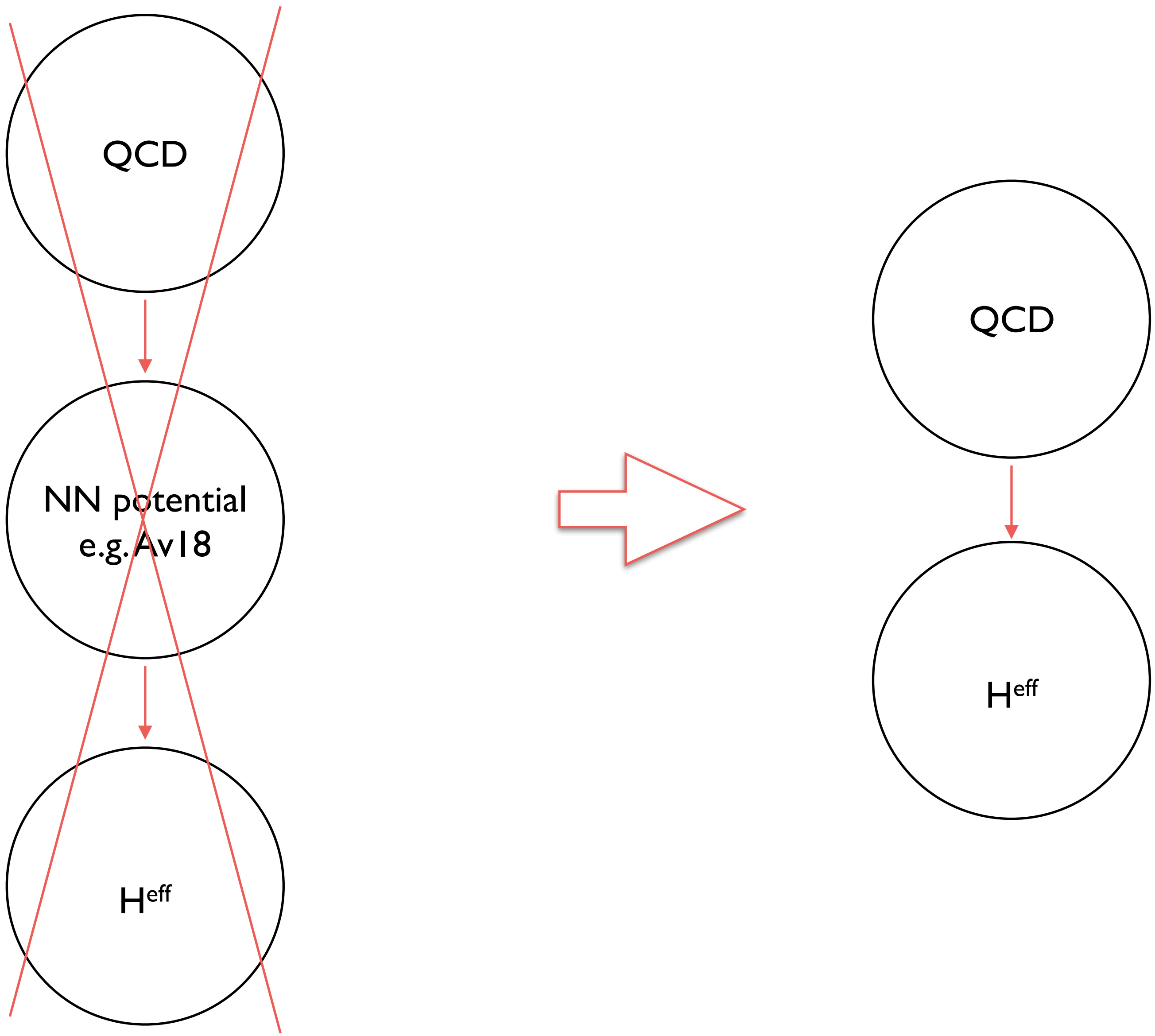
- TRIUMF, RIKEN, MSU, … have built/are building new facilities to manufacture rare isotopes, to probe toward the drip line
  - — is this nuclei collecting, like butterfly collecting?
  - — or is there something special about all of NP that we can learn by studying the limit of weak binding?

Can we re-engineer our approach to nuclear physics?

Deconstruct and reconstruct: take all of the existing parts,
hook them up differently,
to see if a better theory exists

From a modern perspective (ET) , NP is weird





A true nonrelativistic ET

What is an effective theory?

Full Theory:

$$H|\psi\rangle = E|\psi\rangle$$

Effective Theory:

$$PH^{\text{eff}}P|\psi\rangle = EP|\psi\rangle$$

$$P + Q = 1$$

$$|\psi\rangle = P|\psi\rangle + Q|\psi\rangle$$

what do we choose for P?

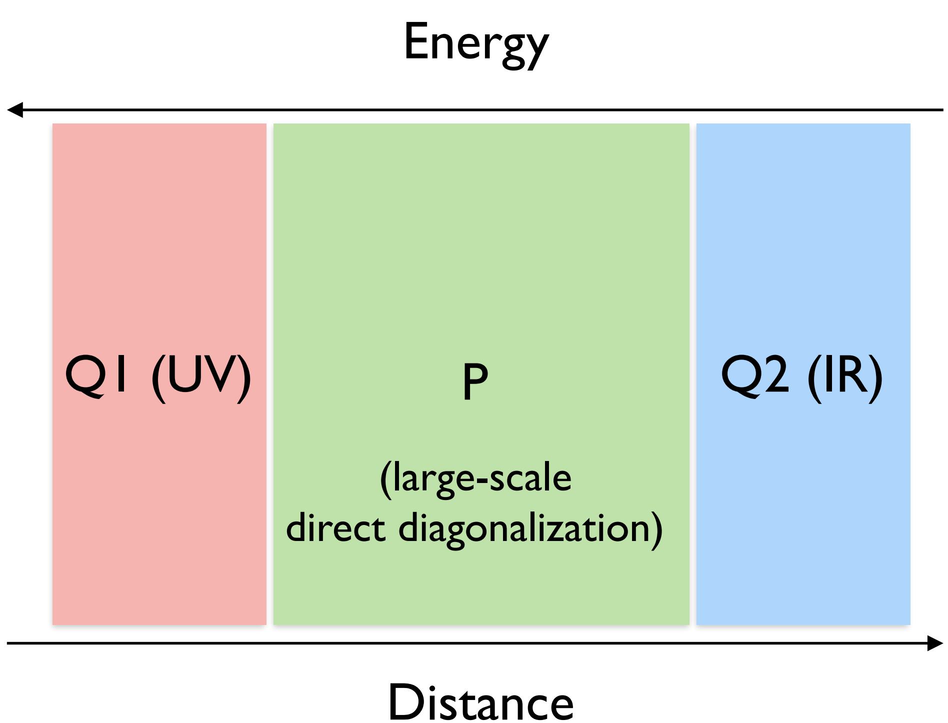
translational invariance: two choices

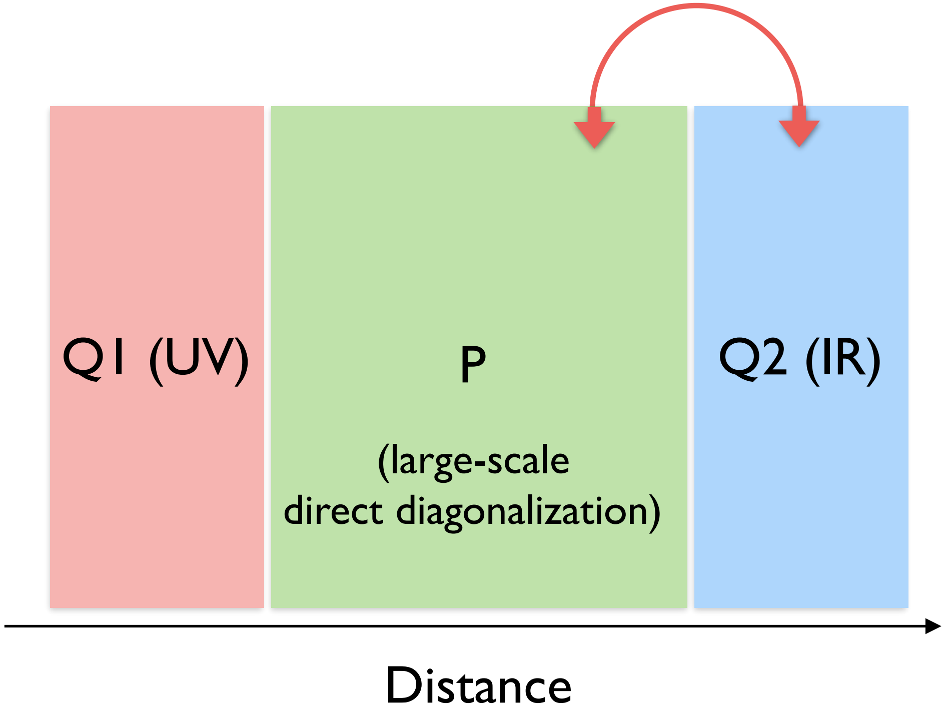
discrete (so we can NP numerical technology): the harmonic oscillator (compact)

The Requires One to Understanding the Functional Form of the H^{eff}

- Nuclear ground states are a compromise between the UV and the IR: kinetic energy is minimized by delocalization; potential by localizing at the strong interaction scale

- This is the essential physics of the ET: corrections due to omitted IR and UV physics are roughly comparable in importance





Coupling between P and Q2 is via the K.E. operator

$\vec{\nabla}^2$ connects neighboring shells

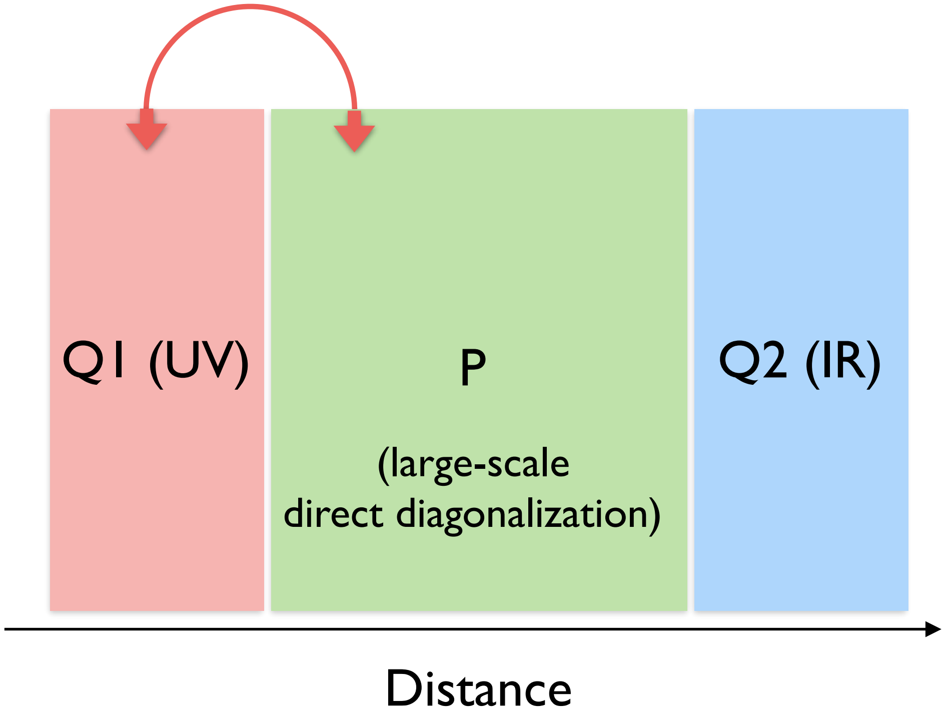
this means small energy denominators, highly energy dependent corrections

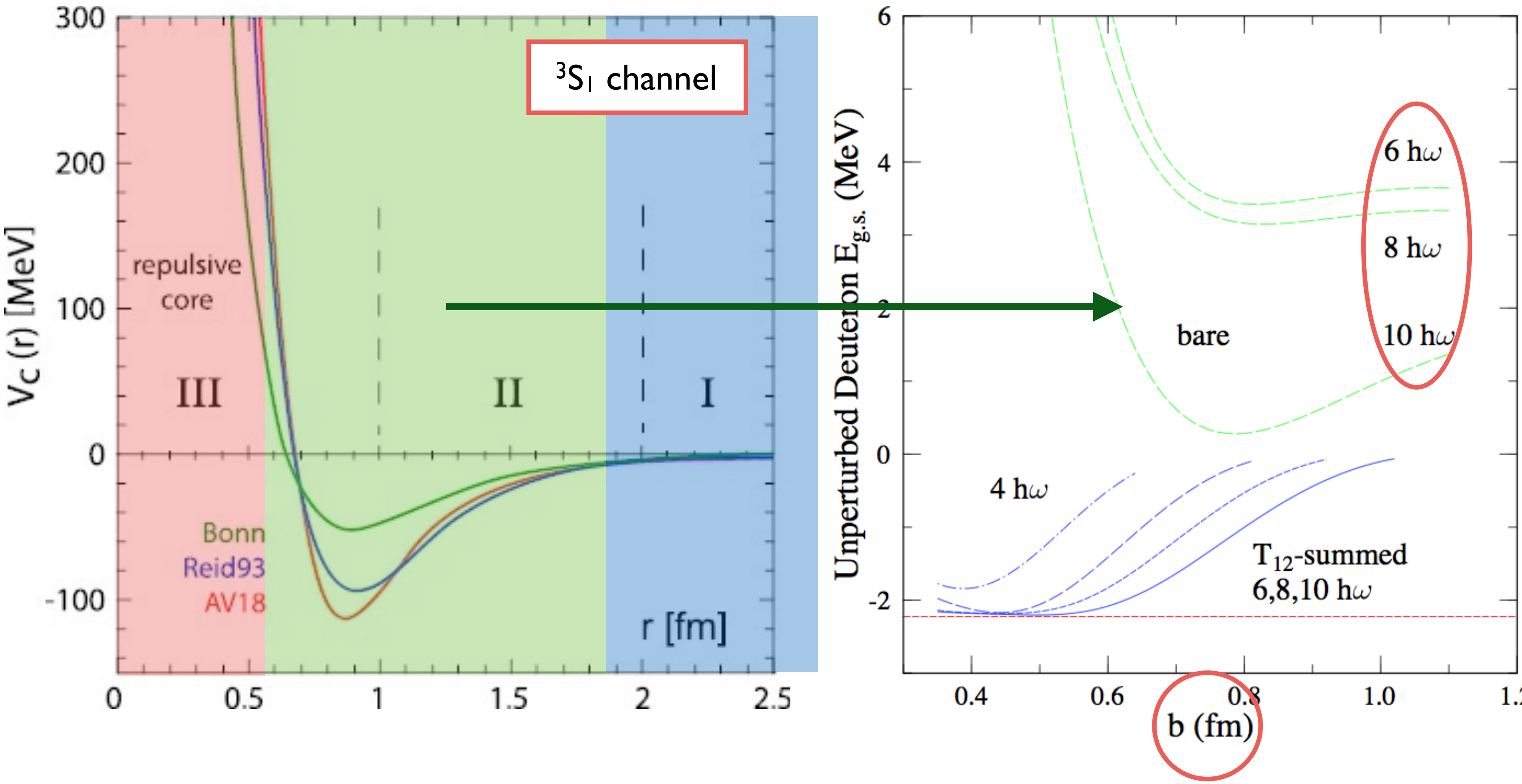
must be treated - but can be quasi-analytically

Coupling between P and Q2 is via short-range strong interactions

Large energy denominators: energy independent corrections

Can be treated by a standard short range expansion

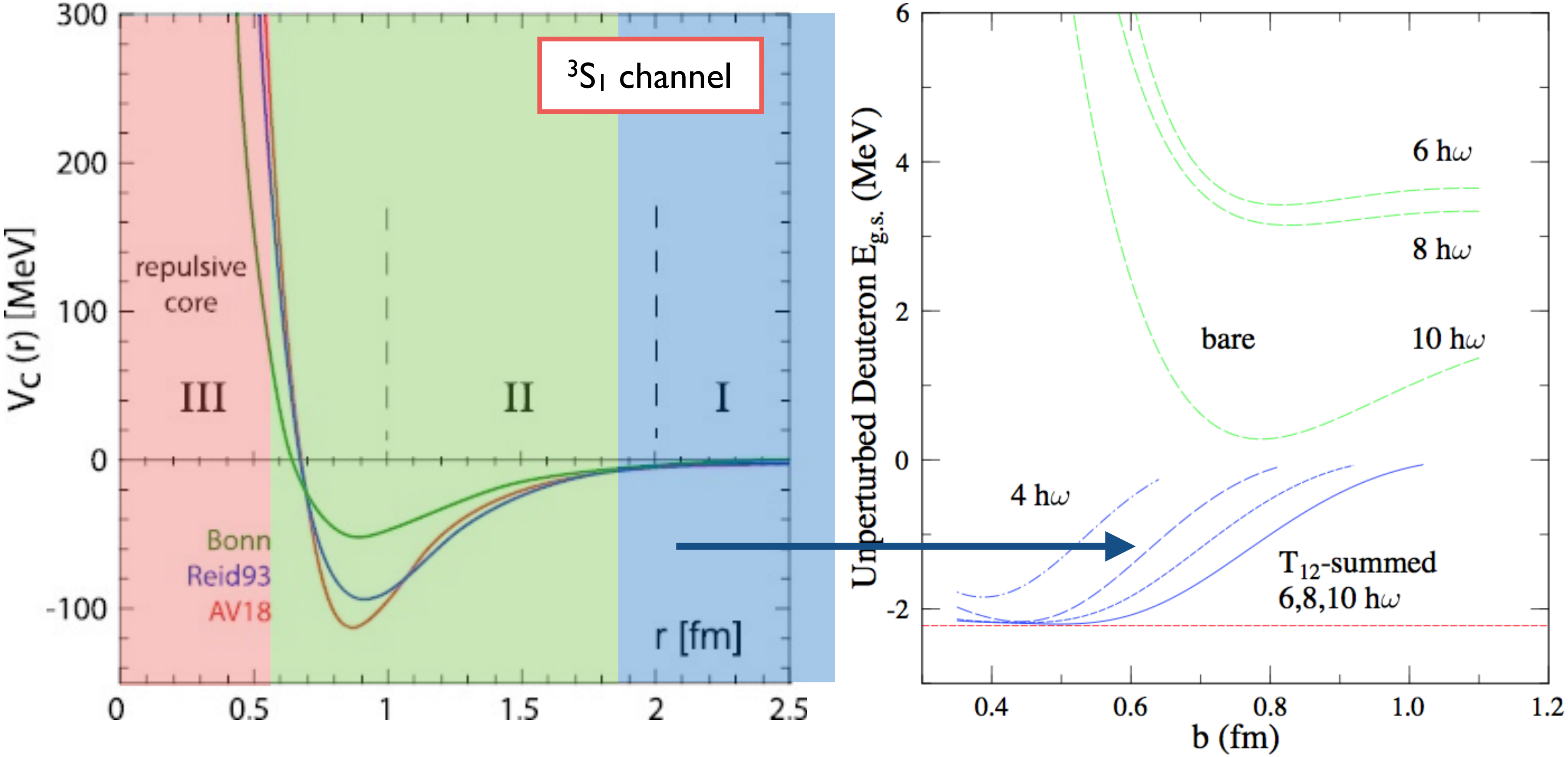




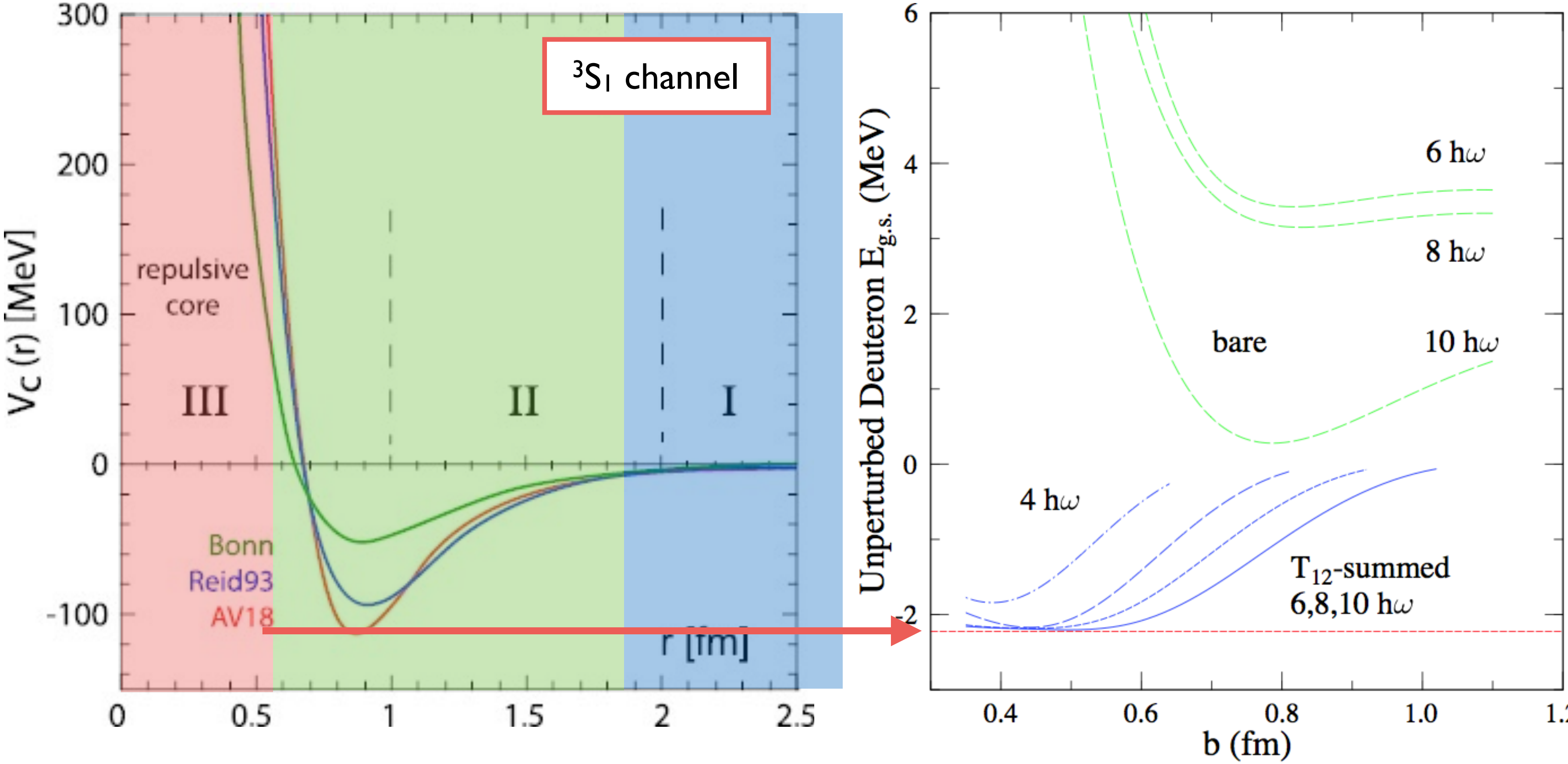
Simple example: the deuteron with av18 potential

standard C.I. approach requires ~100 $\hbar\omega$ to achieve 1 keV accuracy

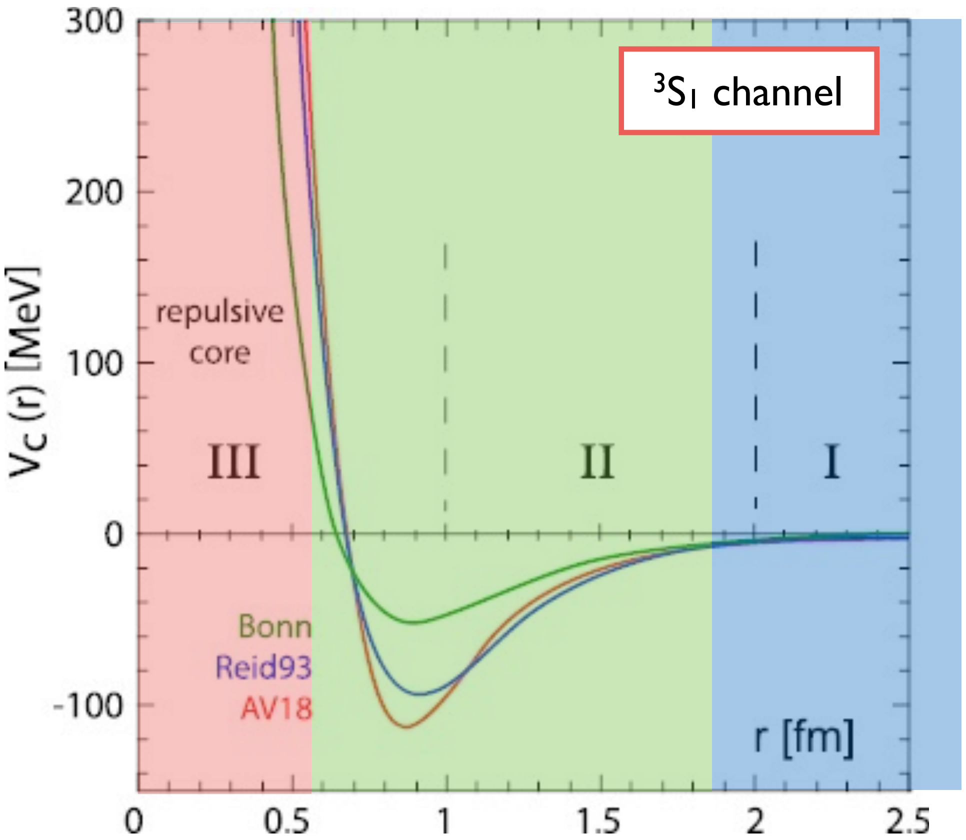
work done by C.-L. Song, Tom Luu

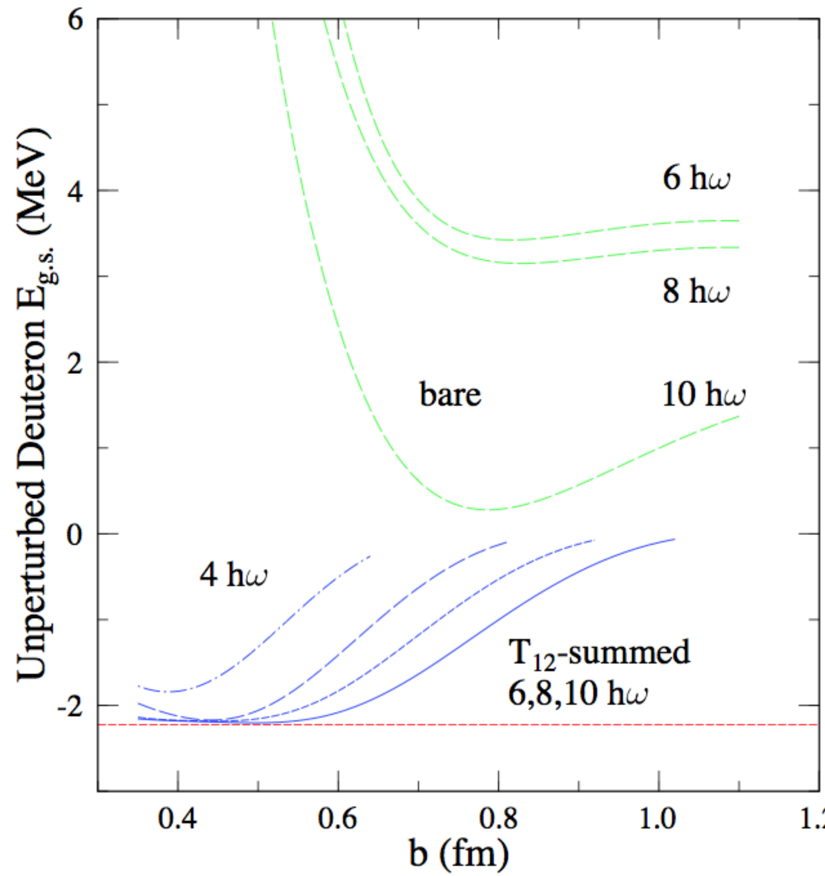


add HOBET's infrared summation



Once the theory has been made "infra-red correct", an rapidly convergent short-range ET yields the exact result independent of the choice of b or Λ





$$H^{\mathrm{eff}} = PH\frac{1}{E-QH}QHP \to P\frac{E}{E-TQ}\left[T - T\frac{Q}{E}T + V + V\frac{1}{E-QH}QV\right]\frac{E}{E-QT}P$$

SOLVE SELF-CONSISTENTLY IN E: WH/Tom Luu Form of the BH Equation

$$H^{\text{eff}} = PH\frac{1}{E-QH}QHP \to P\frac{E}{E-TQ}\left[T - T\frac{Q}{E}T + V + V\frac{1}{E-QH}QV\right]\frac{E}{E-QT}P$$

$$\left[P\frac{E}{E-T}P\right]^{-1}\frac{E}{E-T}$$

cost of a matrix inversion:
(highly E dependent)

analytic

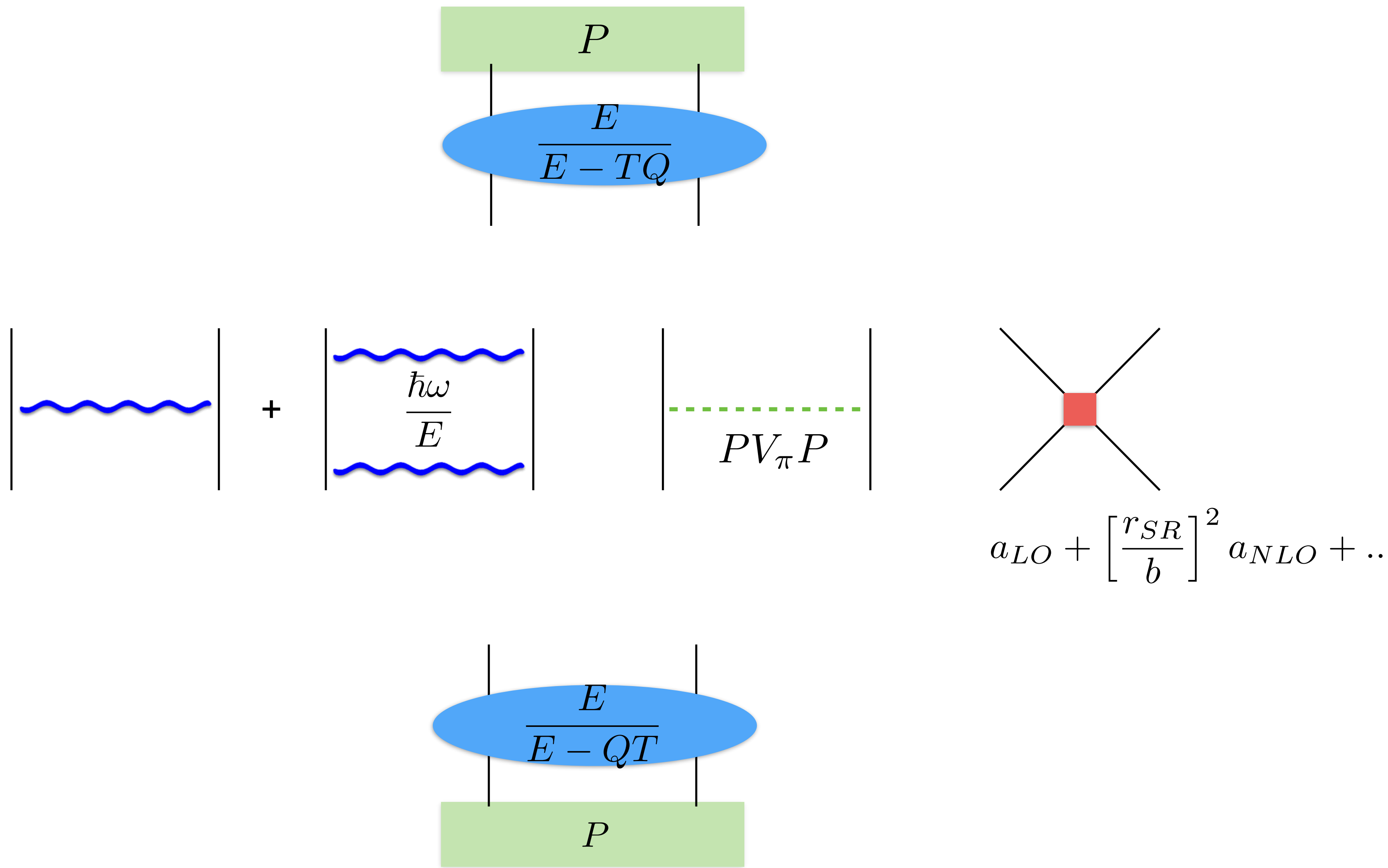
$$H^{\text{eff}} = PH\frac{1}{E-QH}QHP \rightarrow P\frac{E}{E-TQ}\left[T - T\frac{Q}{E}T + V + V\frac{1}{E-QH}QV\right]\frac{E}{E-QT}P$$

$$a_{LO}^{3S1}\delta(\vec{r}) + a_{NLO}^{3S1}(\overleftarrow{\nabla}^2_{HO}\delta(\vec{r}) + \delta(\vec{r})\overrightarrow{\nabla}^2_{HO}) + \cdots$$

second quantized form:
HO raising/lowering
operators

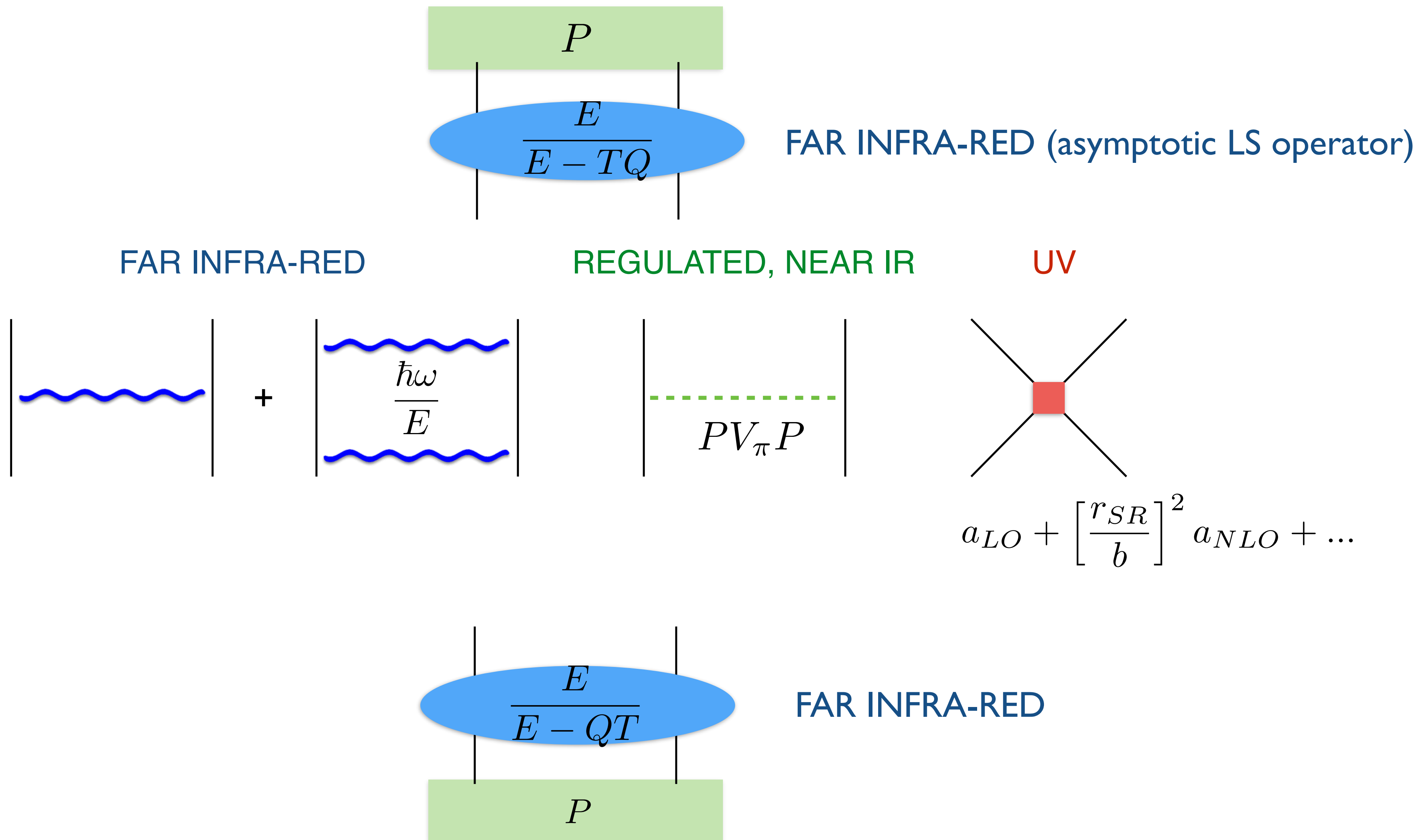
$$\sim a_{LO}^{3S1}|n'\ell = 0\rangle\langle n\,\ell = 0| +$$

$$a^{3S1}{}_{NLO}\left(\boldsymbol{a}^\dagger \odot \boldsymbol{a}^\dagger |n'\ell = 0\rangle\langle n\ell = 0| + |n'\ell = 0\rangle\langle n\,\ell = 0|\tilde{\boldsymbol{a}} \odot \tilde{\boldsymbol{a}}\right) \;+\; \cdots$$



the <u>correct</u> chiral interaction for the HO “SM” - not the form folks use

rapidly convergent


P
E / (E − TQ)
FAR INFRA-RED (asymptotic LS operator)
FAR INFRA-RED
REGULATED, NEAR IR
UV
+
ħω / E
$PV_\pi P$
$a_{LO} + \left[\frac{r_{SR}}{b}\right]^2 a_{NLO} + \ldots$
E / (E − QT)
FAR INFRA-RED
P

CONTRAST WITH WHAT IS USED IN THE BEST MODERN TREATMENTS OF STRUCTURE

LO

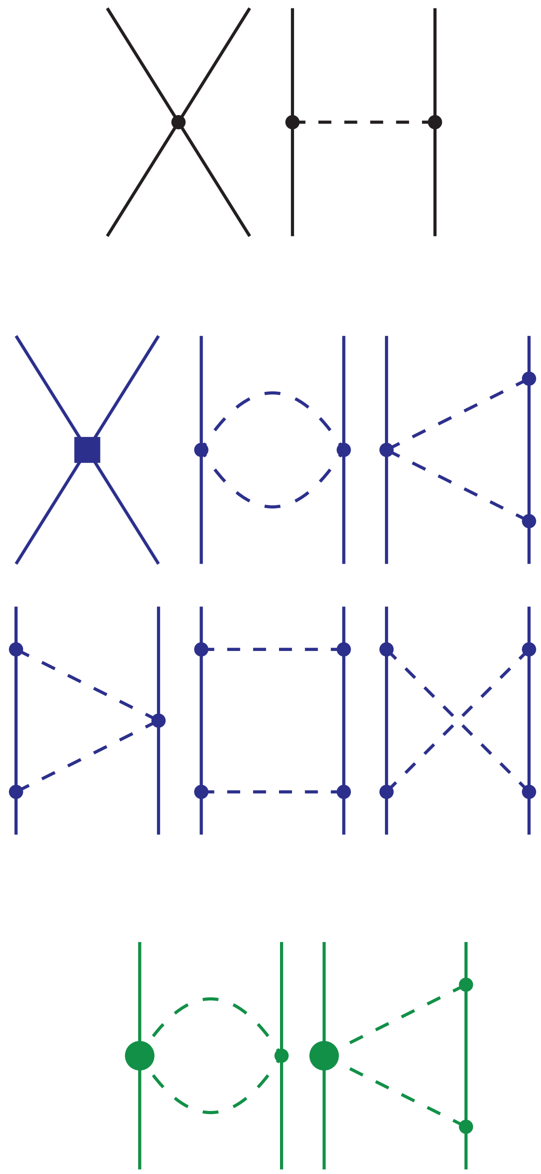
$(Q/\Lambda_\chi)^0$

NLO

$(Q/\Lambda_\chi)^2$

NNLO

$(Q/\Lambda_\chi)^3$

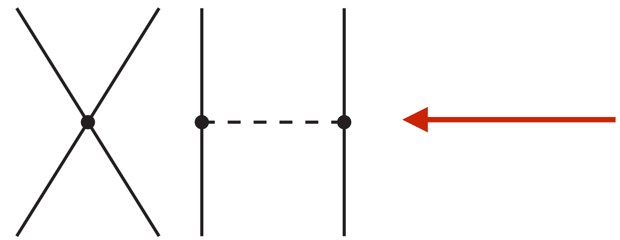


+...

CONTRAST WITH WHAT IS USED IN THE BEST MODERN TREATMENTS OF STRUCTURE

LO

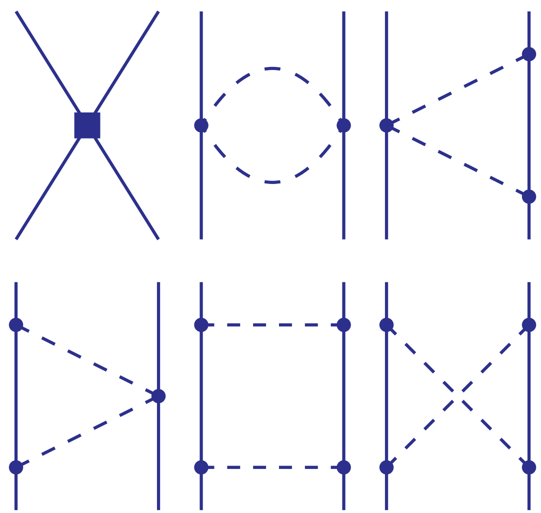
$(Q/\Lambda_\chi)^0$



wrong tensor-force regulator

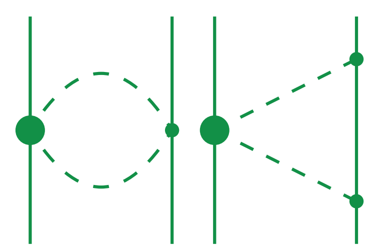
NLO

$(Q/\Lambda_\chi)^2$



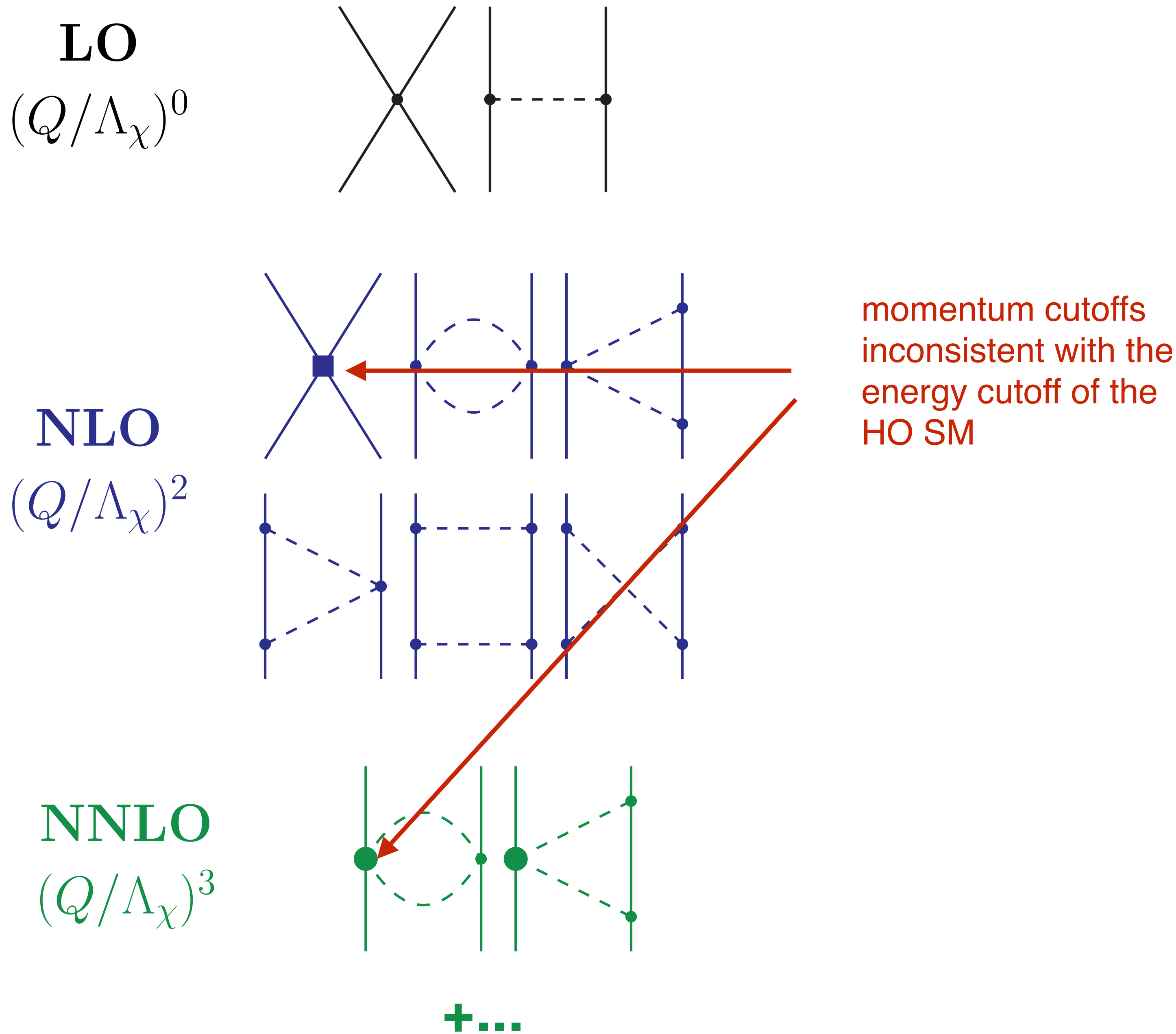
NNLO

$(Q/\Lambda_\chi)^3$



+...

CONTRAST WITH WHAT IS USED IN THE BEST MODERN TREATMENTS OF STRUCTURE

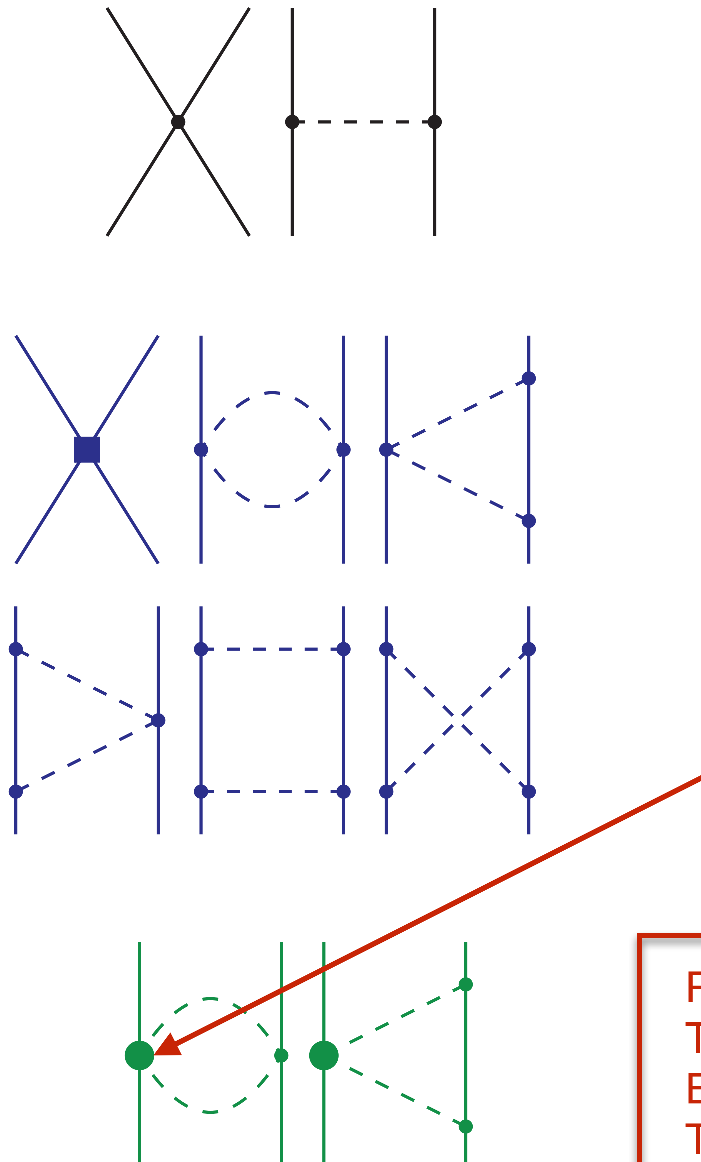


CONTRAST WITH WHAT IS USED IN THE BEST MODERN TREATMENTS OF STRUCTURE

LO
$(Q/\Lambda_\chi)^0$

NLO
$(Q/\Lambda_\chi)^2$

NNLO
$(Q/\Lambda_\chi)^3$



+...

cutoffs inconsistent
with the translational
invariance of the HO

PARAMETERS TUNED TO REPRODUCE ENERGIES, BUT EASY TO SEE PROJECTED WAVE FUNCTIONS ARE WRONG

Love the energy dependence: it is your best friend

- it is a theorem that no short-range effective theory can succeed if one fails to build in the correct IR (long-distance) behavior

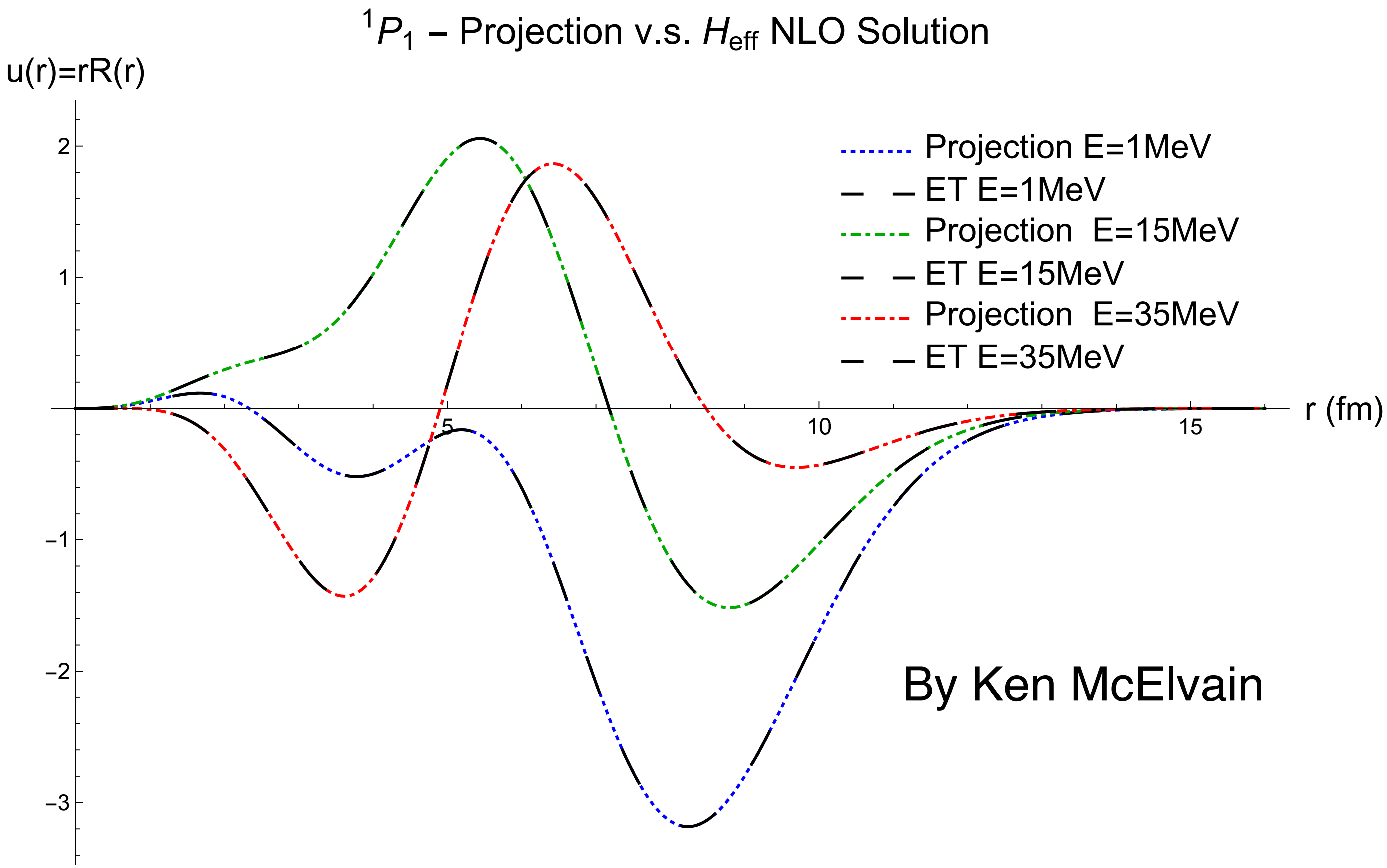
- Chiral HOBET is explicitly analytic in E: it is a seamless and exact ET of both bound states and reactions

- it thus produces the exact restriction of states to the HO, for E<0 & E>0

* IR Green's functions for bound states are determined by E
Thus solving the BH equation self consistently yields eigenvalues

* Continuum solutions exist at every E>0, while their IR Green's functions depend on E and $\delta(E)$

Pick E, look up $\delta(E)$, diagonalize: if the eigenvalue fails to = E, the only unused degree of freedom is UV - so pick an a_{LO}. Repeat …



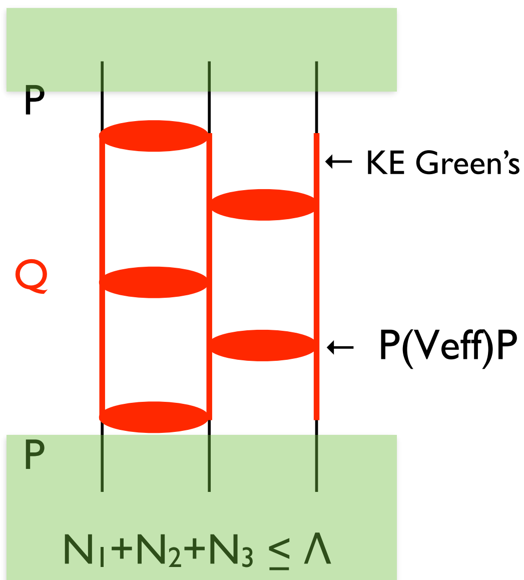
A “SM” interaction has been constructed directly from phase shifts, yielding the exact restriction of the true wave function to P

information previously encoded in, decoded from an NN potential

Also, bound state properties are obtained similarly: from $^3S_1$-$^3D_1$ phase shifts find a deuteron binding energy of -2.2245 MeV

Results are independent of the choices made in defining P

The two-body physics so determined can then be subtracted exactly out of the N-body problem (not yet done):



IR propagators

Interactions are now soft and finite in number: a calculation in an infinite Hilbert space has been reduced to one in a discrete basis

Much left to be done in this “reconstruction” phase, but …

The double beta decay effective interactions problem is essentially identical to that just described…