Effective moments and transition operators in the \textit{ab initio} No-Core Shell Model
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\textbf{The Overarching Questions}
\begin{itemize}
  \item How did visible matter come into being and how does it evolve?
  \item How does subatomic matter organize itself and what phenomena emerge?
  \item Are the fundamental interactions that are basic to the structure of matter fully understood?
  \item How can the knowledge and technological progress provided by nuclear physics best be used to benefit society?
\end{itemize}
- \textit{NRC Decadal Study}

\textbf{The Time Scale}
\begin{itemize}
  \item Protons and neutrons formed $10^{-6}$ to 1 second after Big Bang (13.7 billion years ago)
  \item H, D, He, Li, Be, B formed 3-20 minutes after Big Bang
  \item Other elements born over the next 13.7 billion years
Main Message

Use light nuclei and precision calculations to understand effective operators’ (e.g. $g_A$) dependences on Hamiltonians and basis spaces.

Overview

Review of selected published results and present some new “test” problems.

Conclusion

Major efforts needed to quantify all theoretical uncertainties: Effective Hamiltonians, Effective electroweak operators, Many-body methods, . . . .
No-Core Configuration Interaction calculations


Given a Hamiltonian operator

\[ \hat{H} = \sum_{i<j} \frac{(\vec{p}_i - \vec{p}_j)^2}{2m_A} + \sum_{i<j} V_{ij} + \sum_{i<j<k} V_{ijk} + \ldots \]

solve the eigenvalue problem for wavefunction of \( A \) nucleons

\[ \hat{H} \Psi(r_1, \ldots, r_A) = \lambda \Psi(r_1, \ldots, r_A) \]

- Expand eigenstates in basis states \( |\Psi\rangle = \sum a_i |\Phi_i\rangle \)
- Diagonalize Hamiltonian matrix \( H_{ij} = \langle \Phi_j | \hat{H} | \Phi_i \rangle \)
- No-Core CI: all \( A \) nucleons are treated the same
- Complete basis \( \rightarrow \) exact result
- In practice
  - truncate basis
  - study behavior of observables as function of truncation
Nuclei represent strongly interacting, self-bound, open systems with multiple scales – a computationally hard problem whose solution has potential impacts on other fields.

**Question:** What controls convergence/uncertainties of observables?

**Answer:** Characteristic infrared (IR) and ultraviolet (UV) scales of the operators.

In a plane-wave basis:
- $\lambda =$ lowest momentum scale - can be zero (e.g. $T_{\text{rel}}$, $r^2$, $B(\text{EL})$, . . . )
- $\Lambda =$ highest momentum scale - can be infinity (e.g. $T_{\text{rel}}$, hard-core $V_{NN}$)

In a harmonic-oscillator basis with $N_{\text{max}}$ truncation:

$$\lambda \approx \sqrt{\frac{\hbar \Omega}{N_{\text{max}}}}$$

$$\Lambda \approx \sqrt{\frac{\hbar \Omega}{N_{\text{max}}}}$$

**What are examples of the other physically relevant scales in nuclear physics?**
- Interaction scales (total binding, Fermi momentum, SRCs, one-pion exchange, . . . )
- Leading dissociation scale (halos, nucleon removal energy, . . . )
- Collective motion, clustering scales ($Q$, $B$(E2), giant modes, . . . )
Guidelines for many-body calculations to guarantee preserved predictive power:

1. Select basis regulators:

\[ \lambda \leq \text{all relevant IR scale limits} \]

\[ \Lambda \geq \text{all relevant UV scale limits except } T_{\text{rel}} \]

2. Since \( T_{\text{rel}} \) has simple IR and UV asymptotics, extrapolation is feasible for observables where \( T_{\text{rel}} \) dominates

\[ \begin{align*}
\therefore & \quad \text{J-matrix for scattering – takes both IR and UV limits of HO basis} \\
& \quad \text{IR extrapolation tools developed over past } \sim 5 \text{ years}
\end{align*} \]
Phenomeological NN interaction: JISP16

JISP16 tuned up to $^{16}$O

- Constructed to reproduce $np$ scattering data
- Finite rank separable potential in H.O. representation
- Nonlocal $NN$-only potential
- Use Phase-Equivalent Transformations (PET) to tune off-shell interaction to
  - binding energy of $^3$H and $^4$He
  - low-lying states of $^6$Li (JISP6, precursor to JISP16)
  - binding energy of $^{16}$O

Realistic nuclear Hamiltonian: Ab exitu approach

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**Ground state energy of p-shell nuclei with JISP16**

Compare theory and experiment for 24 nuclei  
Maris, Vary, IJMPE22, 1330016 (2013)

- $^{10}$B – most likely JISP16 produces correct $3^+$ ground state, but extrapolation of $1^+$ states not reliable due to mixing of two $1^+$ states
- $^{11}$Be – expt. observed parity inversion within error estimates of extrapolation
- $^{12}$B and $^{12}$N – unclear whether gs is $1^+$ or $2^+$ (expt. at $E_x = 1$ MeV) with JISP16
Excitation spectrum narrow states in good agreement with data
Ground state magnetic moments with JISP16

Compare theory and experiment for 22 magnetic moments  Maris, Vary, IJMPE22, 1330016 (2013)

\[ \mu = \frac{1}{J+1} \left( \langle J \cdot L_p \rangle + 5.586 \langle J \cdot S_p \rangle - 3.826 \langle J \cdot S_n \rangle \right) \mu_0 \]

Good agreement with data, given that we do not have any meson-exchange currents
Extrapolating to the infinite matrix limit i.e. to the “continuum limit”

Results with both IR and UV extrapolations

References:
S.A. Coon, M.I. Avetian, M.K.G. Kruse, U. van Kolck, P. Maris, and J.P. Vary,
E.D. Jurgenson, P. Maris, R.J. Furnstahl, P. Navratil, W.E. Ormand, J.P. Vary,
   Phys. Rev. C 87, 054312(2013); arXiv 1302.5473
S.N. More, A. Ekstroem, R.J. Furnstahl, G. Hagen and T. Papenbrock,
R.J. Furnstahl, S.N. More and T. Papenbrock,
   Phys. Rev. C89, 044301 (2014); arXiv 1312.6876
S. Koenig, S.K. Bogner, R.J. Furnstahl, S.N. More and T. Papenbrock,
   Phys. Rev. C90, 064007 (2014); arXiv 1409.5997
R.J. Furnstahl, G. Hagen, T. Papenbrock and K.A. Wendt,

=> Uncertainty Quantification
Apply new extrapolation method:
D. Odell, T. Papenbrock and L. Patter, arXiv 1512.04851
NCFC results (does not adopt a renormalization)

Experiment-Theory comparison
RMS(Total E) 0.739 MeV (2%)
RMS(Excit’n E) 0.336 MeV (1%)

$G_T^{\text{exp}} = 2.161$ vs $G_T^{\text{thy}} = 2.198(7)$ (2%)
HH+EFT*: Vaintraub, Barnea & Gazit, PRC79,065501(2009); arXiv0903.1048

Solid - JISP16 (bare)
Dotted - Extrap. B

Graph showing the quenching factor \( \frac{M(GT)}{M(GT)} \) for the reaction \( ^6\text{He}(0^+\text{gs}) \rightarrow ^6\text{Li}(1^+\text{gs}) \) as a function of \( N_{\text{max}} \). The graph includes three curves labeled 30, 20, and \( \hbar \Omega = 10 \text{ MeV} \). The solid curves correspond to the JISP16 NN interaction.
Now compare decay of $^6\text{He}$ with GT decay of two neutrons in a HO trap to a neutron-proton pair in the same trap.

In other words, take away all the many-body correlations but keep mean field effects on the NN correlations.

Examine that quenching as a function of the HO trap and the $N_{\text{max}}$ of the HO basis and compare with the full quenching in $^6\text{He}$. 
Quenching Factor = \( M(GT)_{inf} \) / \( M(GT) \)

- \( h\Omega = 10 \text{ MeV} \)
- Preliminary

**JISP16 NN interaction**

- Solid curves - NN transition in a trap
- \( nn \ (^{1}S_{0}) \rightarrow np \ (^{3}S_{1}) \)
- Dashed curves - NCSM calculations
- \( ^{6}\text{He}(0^{+gs}) \rightarrow ^{6}\text{Li}(1^{+gs}) \)

\[ \Rightarrow \sim \text{half of quenching at } N_{\text{max}} = 0 \text{ due to 2-body correlations that exhibit trends of the full quenching} \]
Guidelines for many-body calculations to guarantee preserved predictive power:

1. Select basis regulators:

\[ \lambda \leq \text{all relevant IR scale limits} \]
\[ \Lambda \geq \text{all relevant UV scale limits except } T_{\text{rel}} \]

2. Since \( T_{\text{rel}} \) has simple IR and UV asymptotics, extrapolation is feasible for observables where \( T_{\text{rel}} \) dominates

\[ \diamond \text{ J-matrix for scattering – takes both IR and UV limits of HO basis} \]
\[ \diamond \text{ IR extrapolation tools developed over past } \sim 5 \text{ years} \]

To follow guideline #1, the OLS method provides the advantage of transforming all operators to act only within the scale fixed by the basis regulators.

**The cost:** induced many-body operators need to be assessed.

**The benefit:** extrapolation may be avoided
With $H$ defining the OLS transformation, same picture applies to other Hermitian operators.
Outline of the OLS process

\[ UHU^\dagger = U[T + V]U^\dagger = H_d \]

\[ H_{\text{eff}} = U_{OLS}HU_{OLS}^\dagger = PH_{\text{eff}}P = P[T + V_{\text{eff}}]P \]

\[ U^P = PUP \]

\[ \tilde{U}^P = P\tilde{U}^P P = \frac{U^P}{\sqrt{U^{P\dagger}U^P}} \]

\[ H_{\text{eff}} = \tilde{U}^{P\dagger}H_d\tilde{U}^P = \tilde{U}^{P\dagger}UHU^\dagger\tilde{U}^P = P[T + V_{\text{eff}}]P \]

\[ O_{\text{eff}} = \tilde{U}^{P\dagger}UOU^\dagger\tilde{U}^P = P[O_{\text{eff}}]P \]

\[ U_{OLS} = \tilde{U}^{P\dagger}U \]
$\nu - ^{12}\text{C}$ cross section
and the $0^+ 0 \rightarrow 1^+ 1$
Gamow-Teller transition
A.C. Hayes, P. Navratil, J.P. Vary,
PRL 91, 012502 (2003); nucl-th/0305072

First successful description
of the GT data required 3NF.
Both CDBonn + TM' or
AV8' + TM' => large enhancement

N3LO+3NF (OLS) results from:
P. Navratil, V.G. Gueorguiev,
J.P. Vary, W.E. Ormand and

$N_{\text{max}} = 6, 8$ results with SRG on
N3LO+3NF (N2LO); P. Maris, et al,
PRC 90, 014314 (2014) [▲]

Non-local NN interaction
from inverse scattering
(JISP16) also successful
Origin of the anomalously long life-time of $^{14}$C

near-complete cancellations between dominant contributions within $p$-shell

very sensitive to details

Maris, Vary, Navratil, Ormand, Nam, Dean,
PRL106, 202502 (2011)
**Comparison GT transitions in A = 14**

![Comparison GT transitions in A = 14](image)

- Chiral 3-body interactions leads to suppression of GT transition for $^{14}\text{C}(0^+, 0)$ state, but not for $^{14}\text{C}^*(0^+, 2)$ state.

Consider two nucleons as a model problem with $V = \text{JISP16}$

$\lambda(\text{JISP16}) \sim 50 \text{ MeV/c} \& \Lambda(\text{JISP16}) \sim 500 \text{ MeV/c}$ solved in the harmonic oscillator basis with $\hbar\Omega = 10, 20 \text{ and } 30 \text{ MeV}$. Also, consider the role of an added harmonic oscillator quasipotential

Hamiltonian #1

$$H = T + V$$

Hamiltonian #2

$$H = T + U_{\text{osc}} (\hbar\Omega_{\text{basis}}) + V$$

Other observables:

- Magnetic dipole moment $M1$
- Root mean square radius $R$
- Electric quadrupole moment $Q$
- Gamow-Teller $\text{GT}$

Dimension of the “full space” is 120 for all results depicted here
OLS manages the IR and UV regions of the interaction

Note that $\Lambda_{NN}^{(JISP16)} \sim 500 \text{ MeV/c}$

OLS results are independent of IR and UV basis regulators
Truncation vs. OLS for B(GT) 1S0→3S1 β-decay

Preliminary

Fractional Difference

\[ N_{\text{Max}} + 1 \]
Calculation of three-body forces at $N^3LO$

**Goal**
Calculate matrix elements of 3NF in a partial-wave decomposed form which is suitable for different few- and many-body frameworks.

**Challenge**
Due to the large number of matrix elements, the calculation is extremely expensive.

**Strategy**
Develop an efficient code which allows to treat arbitrary local 3N interactions.

(Krebs and Hebeler)
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