A path to lepton-nucleus reactions from first principles

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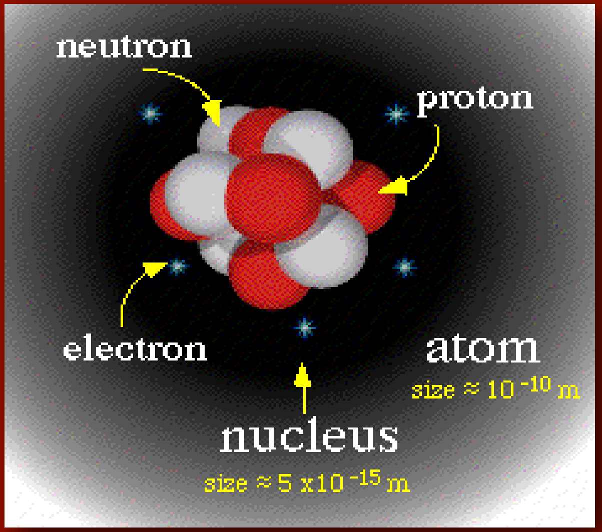
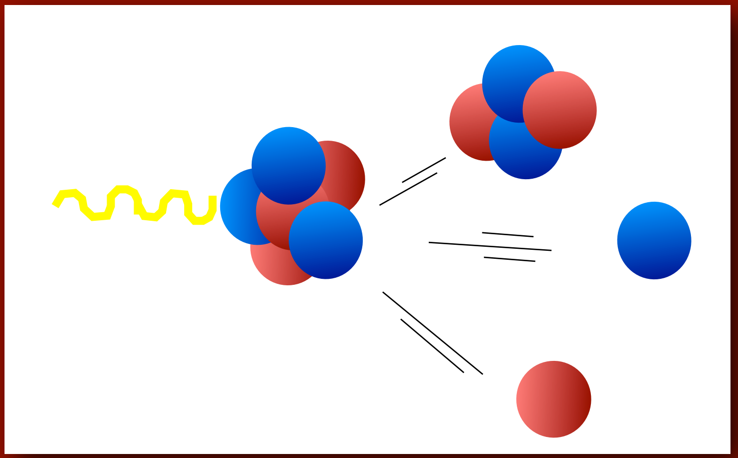
Giuseppina Orlandini, Thomas Papenbrock, Tianrui Xu, et al.

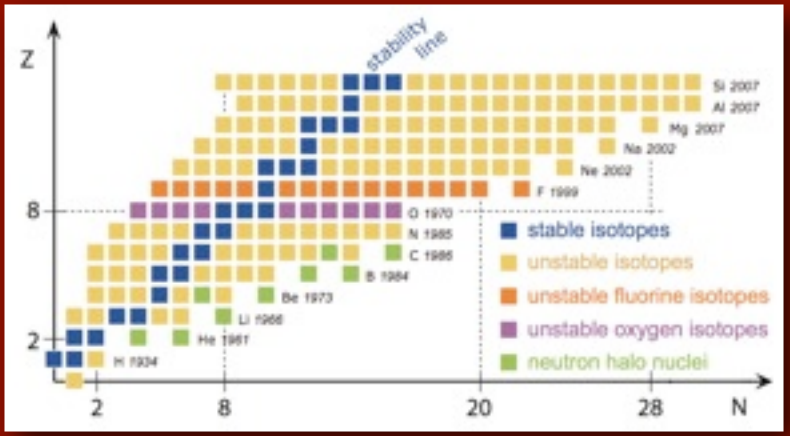
TRIUMF Double Beta Decay Workshop, May 11-13, 2016



The Atomic Nucleus



Electroweak Reactions



Nuclear Chart

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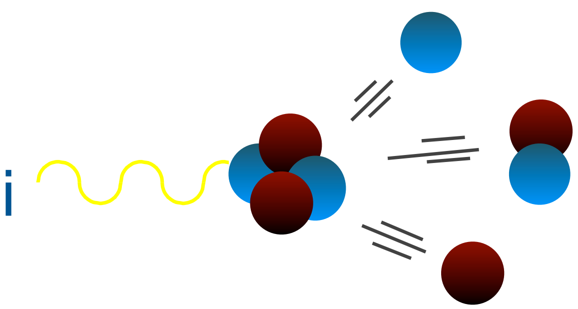
From First Principles

- Describe the nucleus as a system of interacting protons and neutrons and solve the (non-relativistic) many-body problem without approximations or only with controllable approximations.

Aim:

1) Develop a strong predictive theory in the framework of light nuclei where one can test nuclear theory ingredients
2) Then extend it towards heavier systems and/or to observables where experiments are hard



- To provide useful numbers for low-energy nuclear physics:
  - Double beta decay processes
  - interaction of neutrinos with nucleonic matter

Low-energy

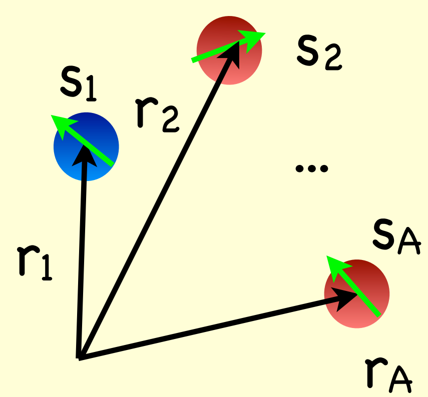
- To provide interpretations for particle physics experiments:
  - neutrino-interactions with nuclei

T2K

Higher-energy

Ab-initio Theory Tools

Nuclear Forces



$$H|\psi_i\rangle = E_i|\psi_i\rangle$$

$$H = T + V_{NN} + V_{3N} + \ldots$$

High precision two-nucleon potentials:
well constraint on NN phase shifts

Three nucleon forces:
less known, constraint on A>2 observables

Traditional Nuclear Physics

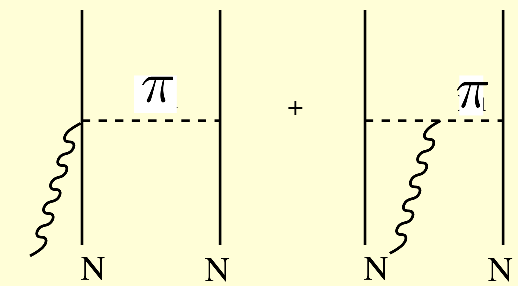
AV18+UIX, ..., J_2

Effective Field Theory

N^2LO, N^3LO ...

Nuclear Currents

$$J^\mu = J^\mu_N + J^\mu_{NN} + \ldots$$



exchange currents

subnuclear d.o.f.

J^μ consistent with V

$$\nabla \cdot J = -i[V, \rho]$$

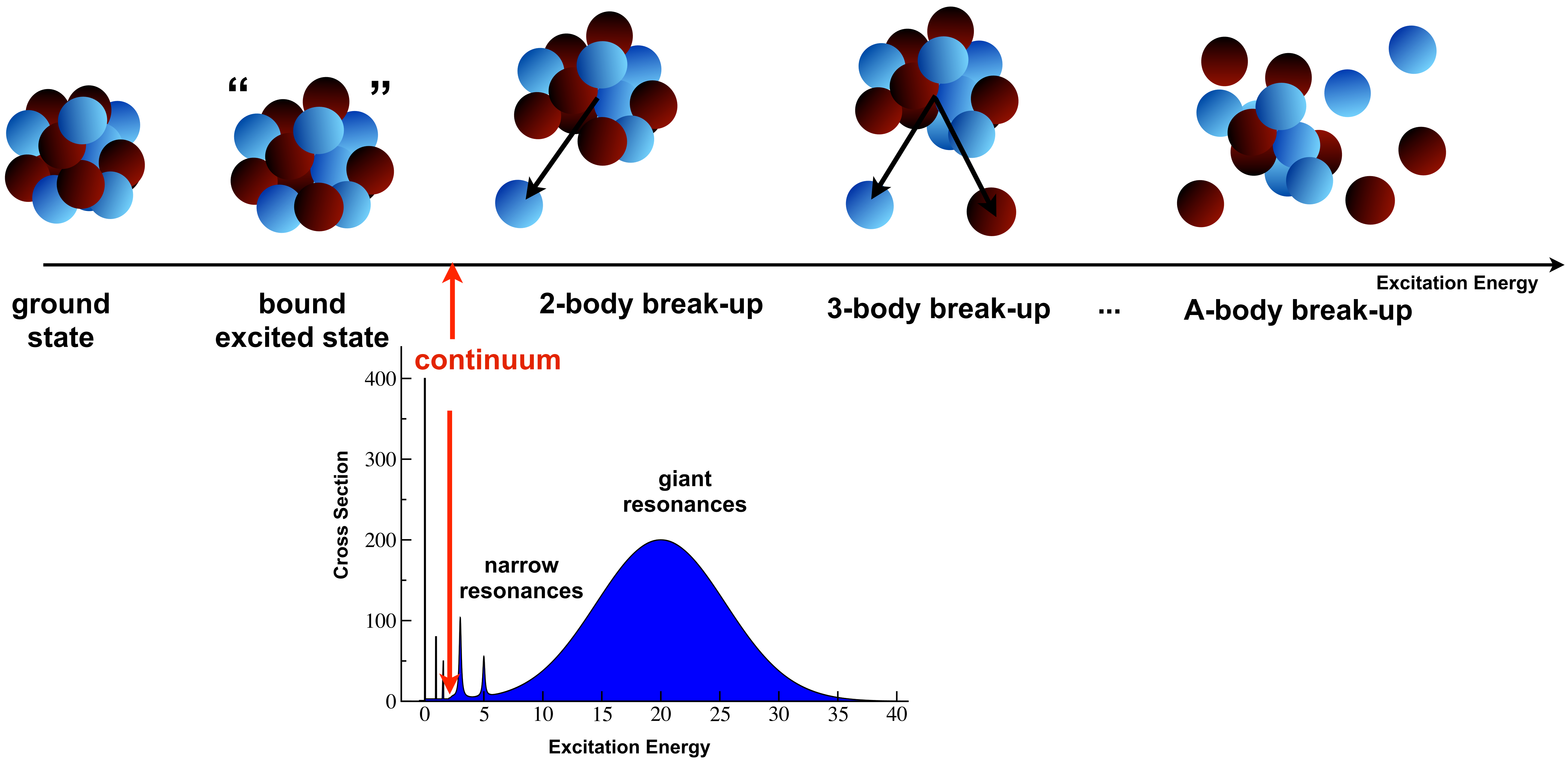
Cross section

$$\sigma \propto \left| \left\langle \psi_f | \hat{J}_\mu | \psi_0 \right\rangle \right|^2$$

Review Paper:
Electromagnetic Reactions on Light Nuclei
S. Bacca and S. Pastore
J. Phys. G: Nucl. Part. Phys. **41** 123002 (2014).

The Continuum Problem

$$\sigma \propto R(q,\omega) \qquad\qquad R(q,\omega) = \sum\!\!\!\!\!\!\!\int_f \left| \left\langle \psi_f | \hat{J}_\mu(q) | \psi_0 \right\rangle \right|^2 \delta(E_f - E_0 - \omega)$$

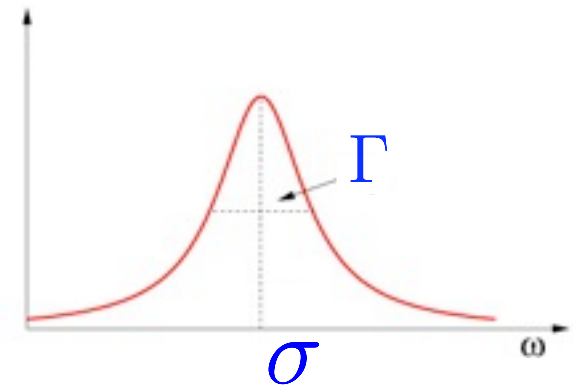


Lorentz Integral Transform

Efros *et al.*, Nucl.Part.Phys. **34** (2007) R459

Response in the continuum

$$R(\omega) = \sum_f \left| \left\langle \psi_f \left| \hat{J}_\mu \right| \psi_0 \right\rangle \right|^2 \delta(E_f - E_0 - \omega)$$



$$L(\sigma,\Gamma) = \int d\omega \frac{R(\omega)}{(\omega-\sigma)^2+\Gamma^2} = \langle\tilde{\psi}|\tilde{\psi}\rangle$$

$$\boxed{(H - E_0 - \sigma + i\Gamma)\,|\,\tilde{\psi}\rangle = \hat{J}_\mu |\,\psi_0\rangle}$$

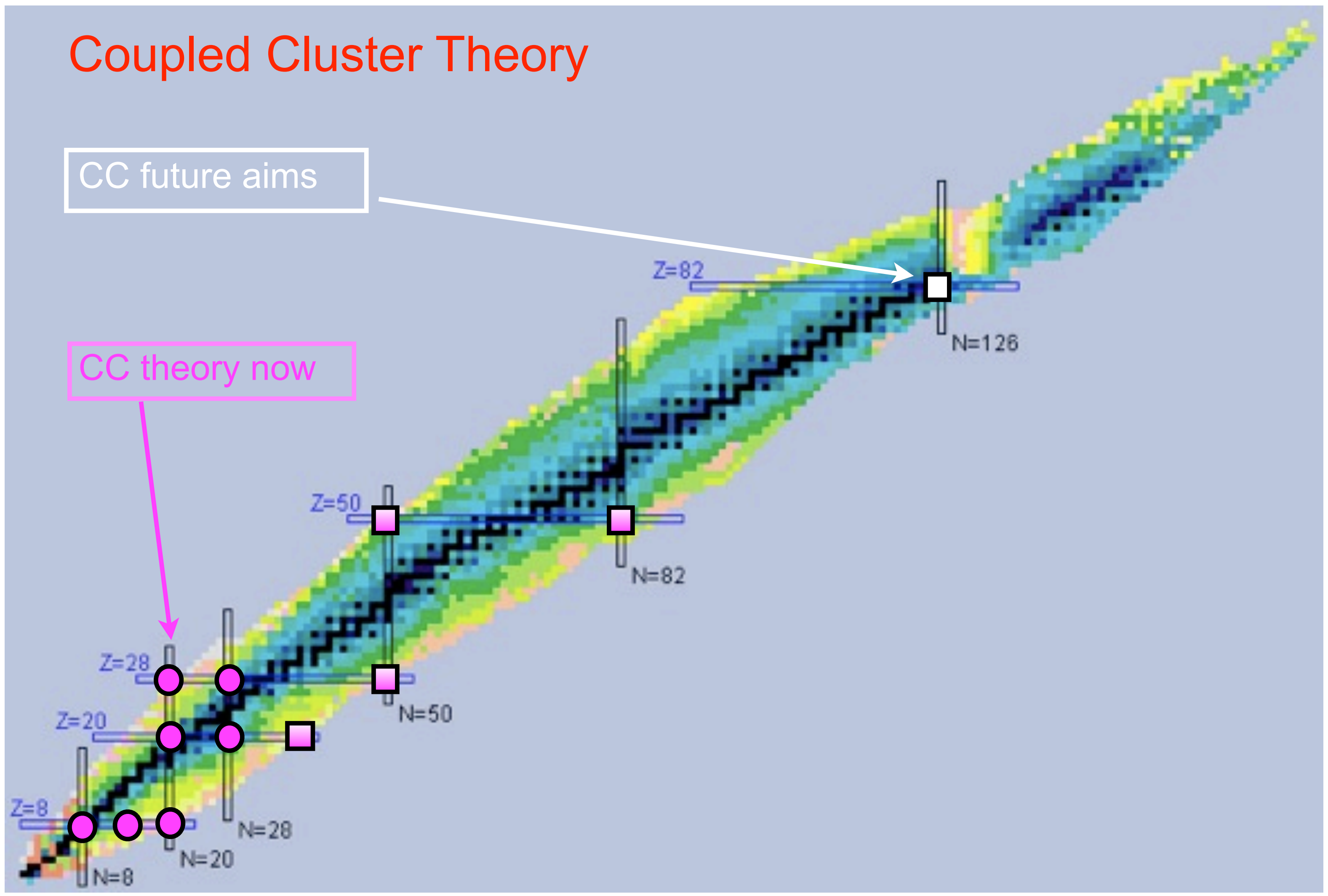
- Due to imaginary part Γ the solution $|\tilde{\psi}\rangle$ is unique
- Since the r.h.s. is finite, then $|\tilde{\psi}\rangle$ has bound state asymptotic behaviour

You can use any good bound state method! e.g. Hyperspherical Harmonics, No Core Shell Model, Coupled Cluster Theory

$$L(\sigma,\Gamma) \xrightarrow{\text{inversion}} R(\omega)$$ with the exact final state interaction

Medium-mass nuclei

Develop new many-body methods that can extend the frontiers to heavier and neutron nuclei



- CC is optimal for closed shell nuclei (±1, ±2)

Uses particle coordinates

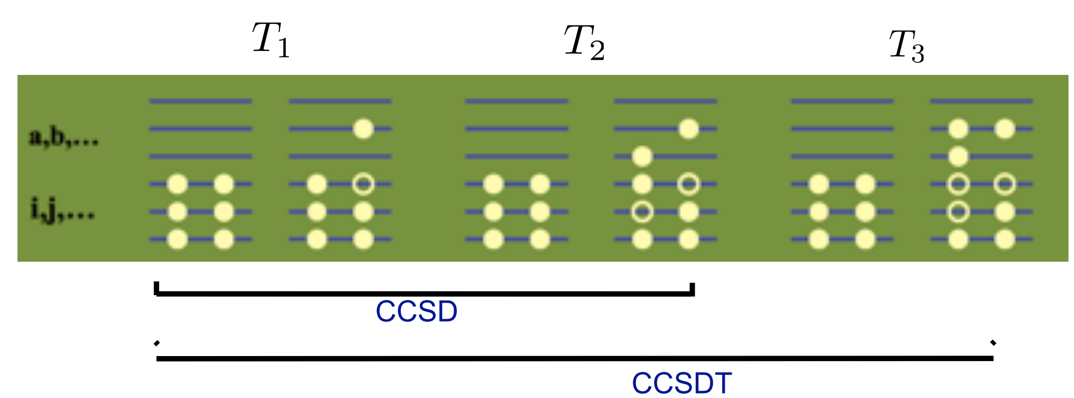
$$|\psi_0(\vec{r}_1, \vec{r}_2, ..., \vec{r}_A)\rangle = e^T |\phi_0(\vec{r}_1, \vec{r}_2, ..., \vec{r}_A)\rangle$$

reference SD with any sp states

$$T = \sum T_{(A)}$$ cluster expansion

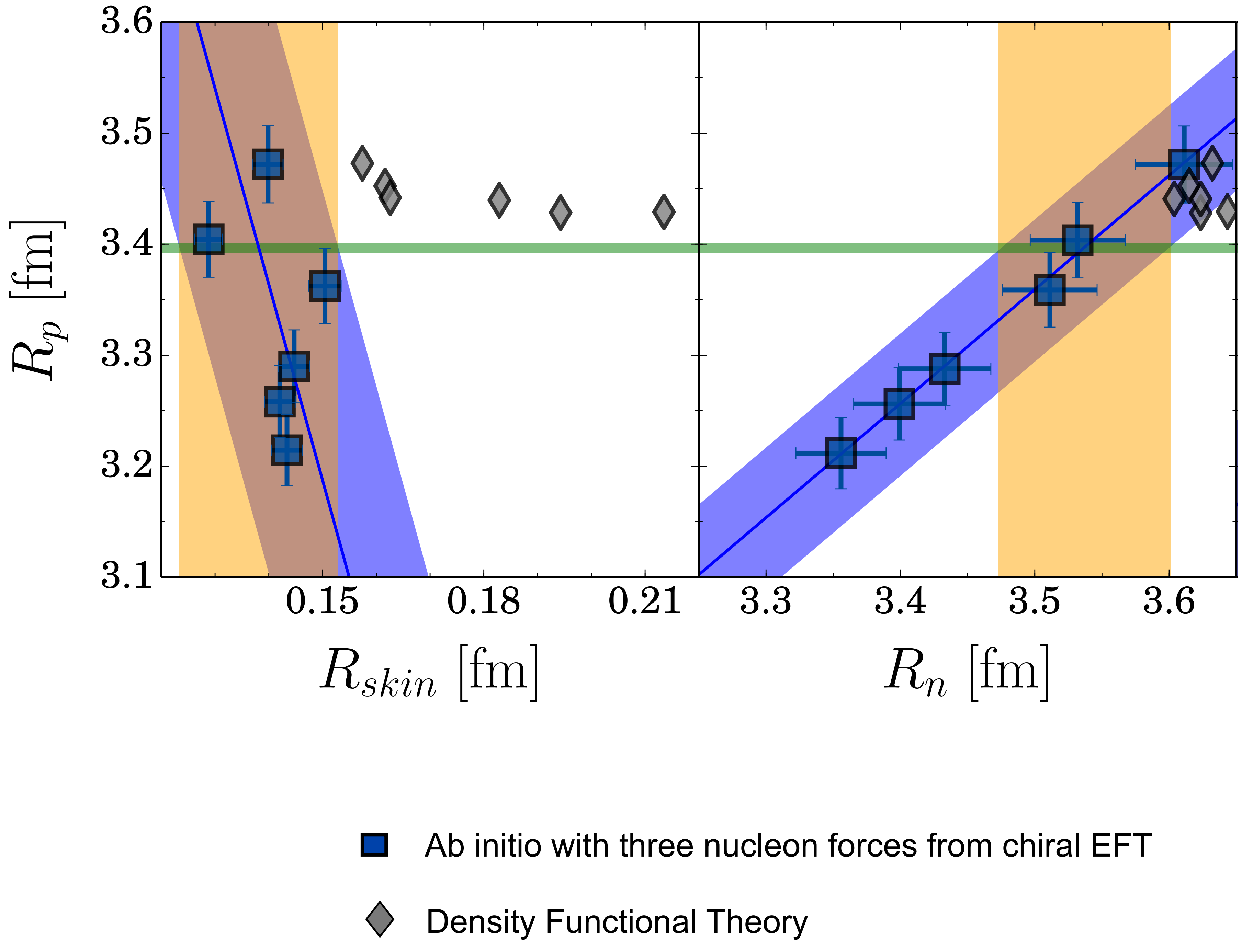
$$T_1 = \sum_{ia} t_i^a a_a^\dagger a_i$$

$$T_2 = \frac{1}{4} \sum_{ij,ab} t_{ij}^{ab} \; a_a^\dagger a_b^\dagger a_j a_i \qquad \cdots$$



^{48}Ca from first principles

Hagen et al., Nature Physics **12**, 186-190 (2016)



LIT with Coupled Cluster Theory

New theoretical method aimed at extending *ab-initio* calculations towards medium-mass

S.B. *et al.*, PRL **111**, 122502 (2013)

$$\boxed{(H - z^*)|\tilde{\Psi}\rangle = J^\mu|\psi_0\rangle} \longrightarrow \boxed{(\bar{H} - z^*)|\tilde{\Psi}_R(z^*)\rangle = \bar{\Theta}|\Phi_0\rangle}$$

with $z = E_0 + \sigma + i\Gamma$

$$\bar{H} = e^{-T} H e^{T}$$

$$\bar{\Theta} = e^{-T} \Theta e^{T}$$

$$L(\sigma,\Gamma) = \left\langle \tilde{\Psi} | \tilde{\Psi} \right\rangle \longrightarrow L(\sigma,\Gamma) = \left\langle \tilde{\Psi}_L | \tilde{\Psi}_R \right\rangle =$$

$$-\frac{1}{2\pi}\Im\left\{\langle \bar{0}_L|\bar{\Theta}^\dagger\left[|\tilde{\Psi}_R(z^*)\rangle - |\tilde{\Psi}_R(z)\rangle\right]\right\}$$

with $|\tilde{\Psi}_R(z^*)\rangle = \hat{R}(z^*)|\Phi_0\rangle$

Formulation based on the solution of an Equation of Motion with source

No approximations done so far!

Present implementation in the CCSD scheme

$$T = T_1 + T_2$$

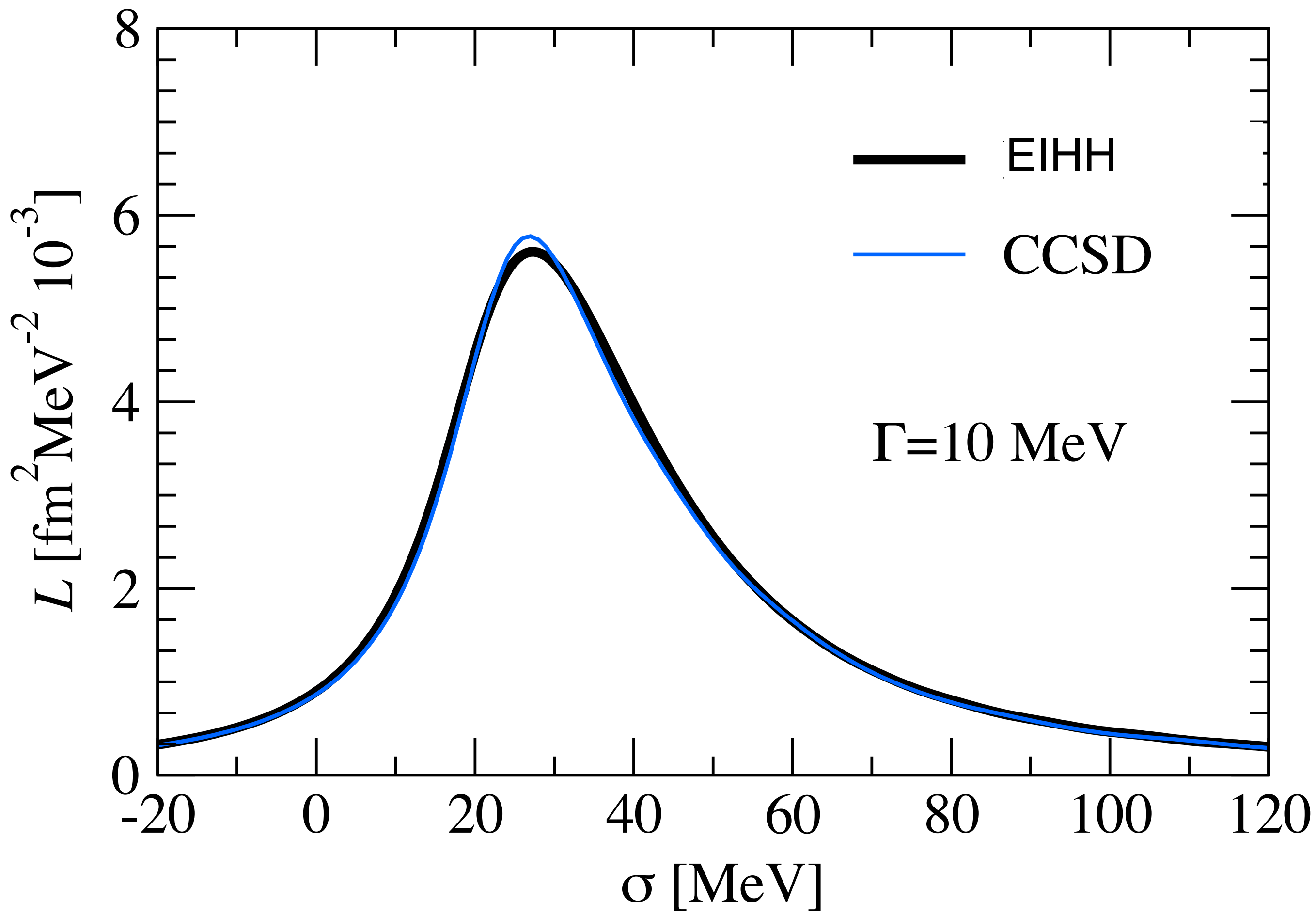
$$\hat{R} = \hat{R}_0 + \hat{R}_1 + \hat{R}_2$$

LIT with Coupled Cluster Theory

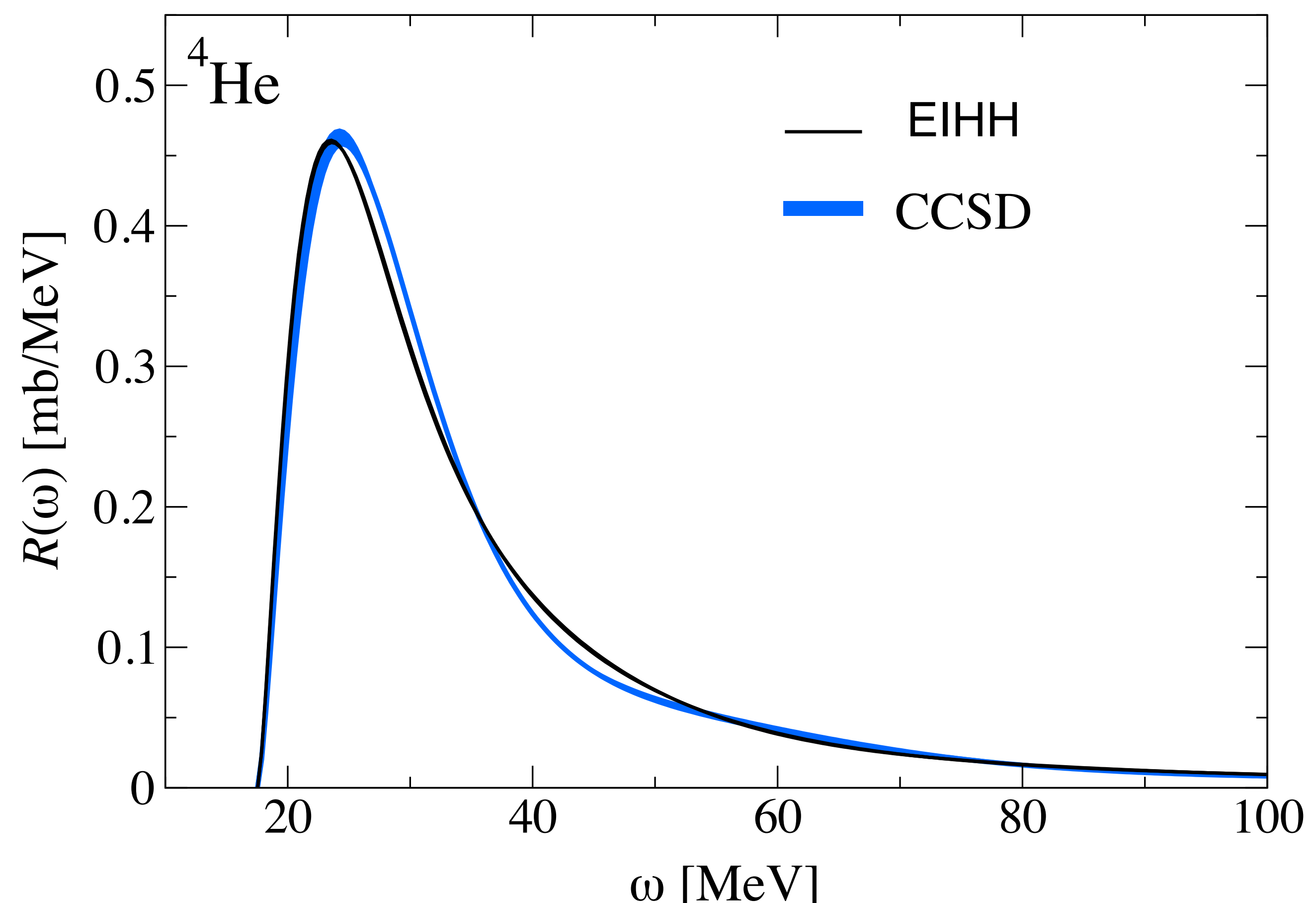
New theoretical method aimed at extending *ab-initio* calculations towards medium mass

Validation for ^{4}He

Comparison of CCSD with exact hyperspherical harmonics (EIHH) with NN forces at N^3LO



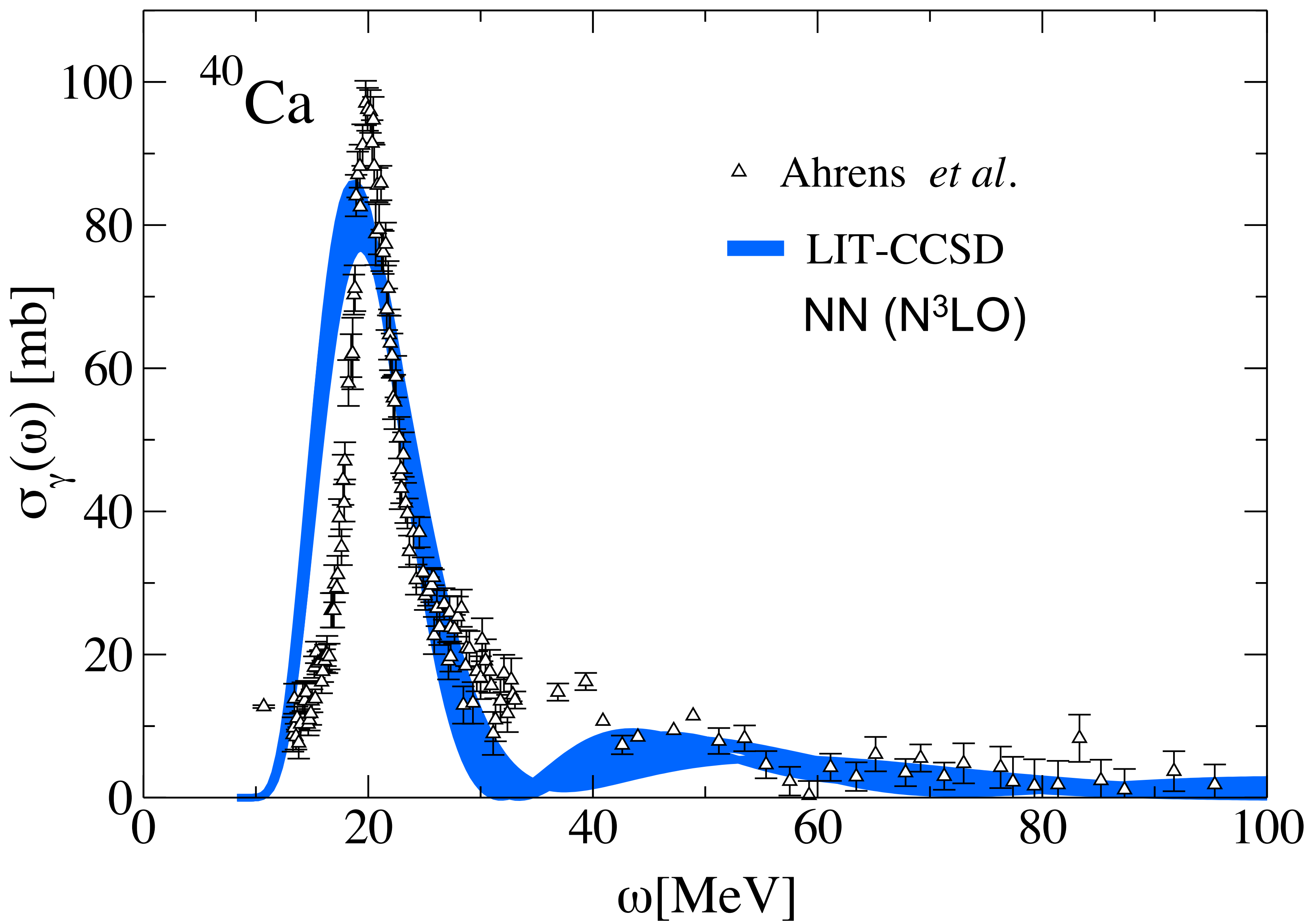
The comparison with exact theory is very good!



Extension to heavier nuclei

With Mirko Miorelli, PhD student at UBC

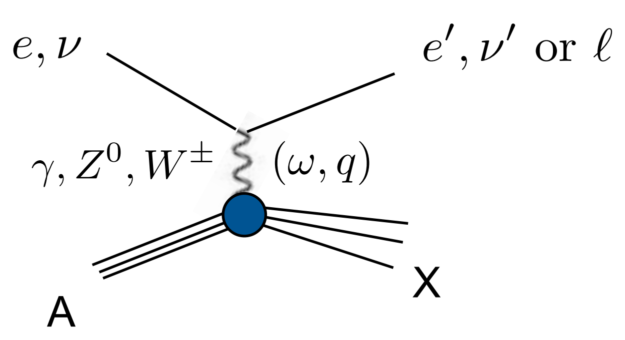
PRC **90**, 064619 (2014)



Neutrino-nucleus cross section

Neutrino long baseline experiments (T2K, Miniboon, LBNE, etc.) require theoretical input to simulate the interaction of neutrinos with the detector material (^{12}C, ^{16}O, ^{40}Ar, ...)

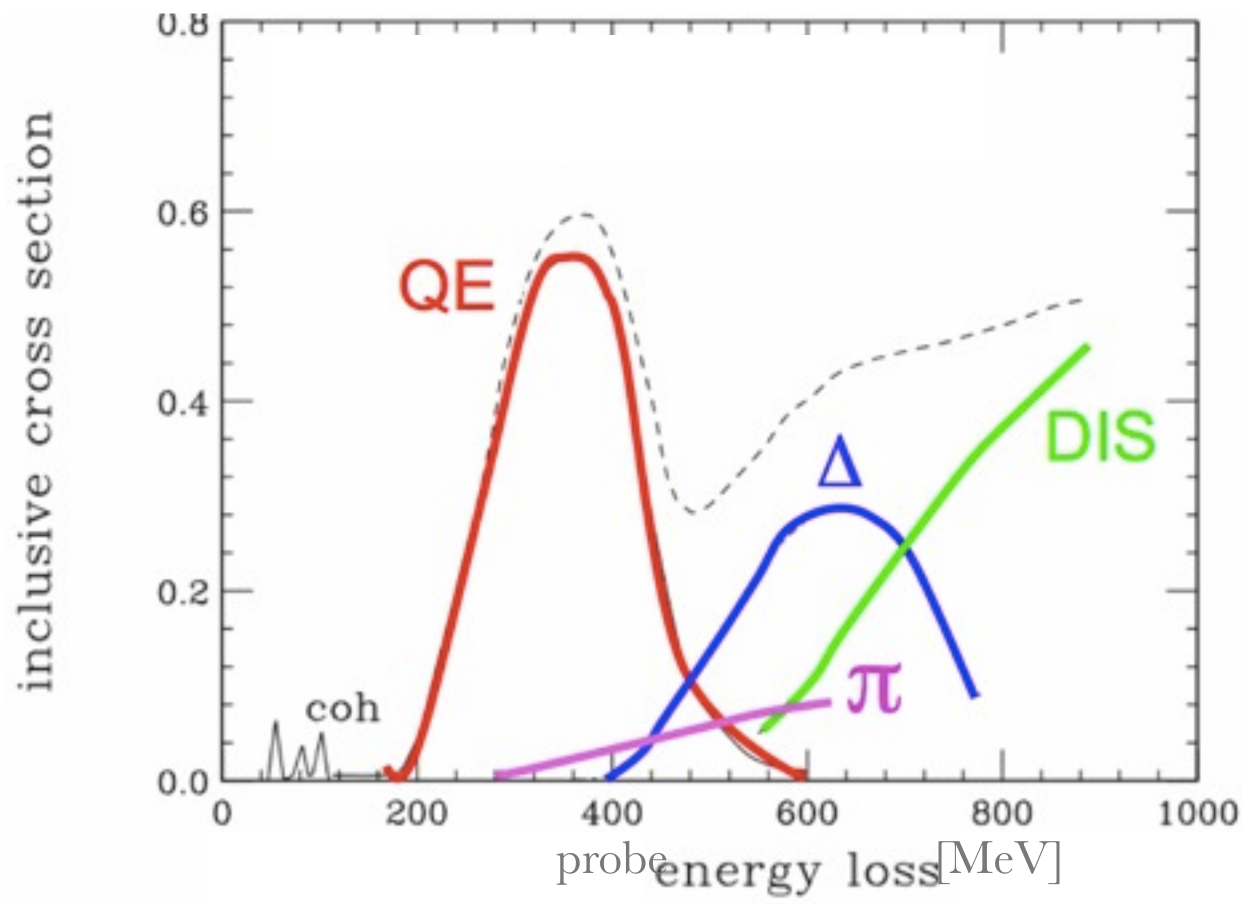
So far very simple models (RMF) are used and ab-initio calculations with reliable error estimates are sought for



- Quasi Elastic part can be studied with the LIT method and coupled-cluster theory

- Need to test theory on electron scattering data first

Neutrino scattering ⇔ electron scattering



Ultimate goal: addressing inclusive neutrino -^{16}O interactions for T2K

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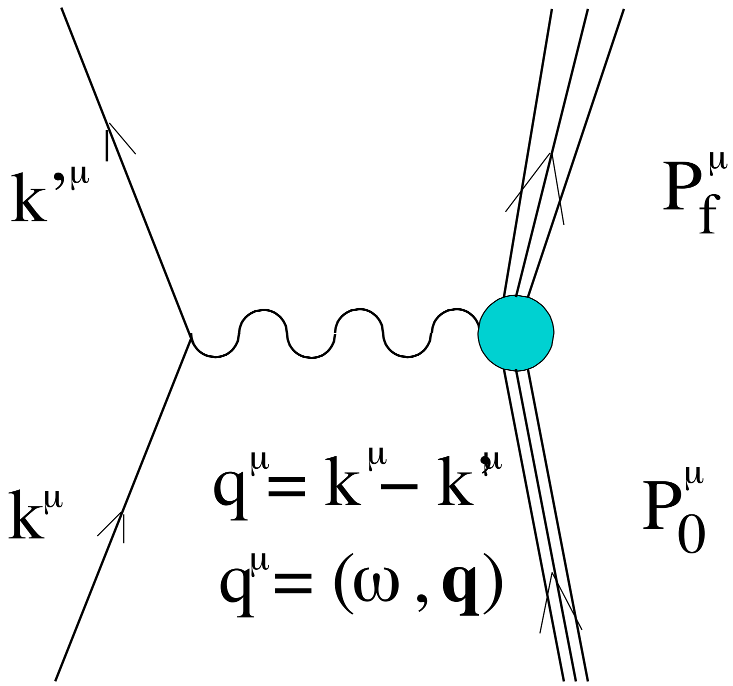
Inelastic e-Scattering A(e,e')X



Virtual Photon

$$(\omega, \mathbf{q})$$

can vary independently

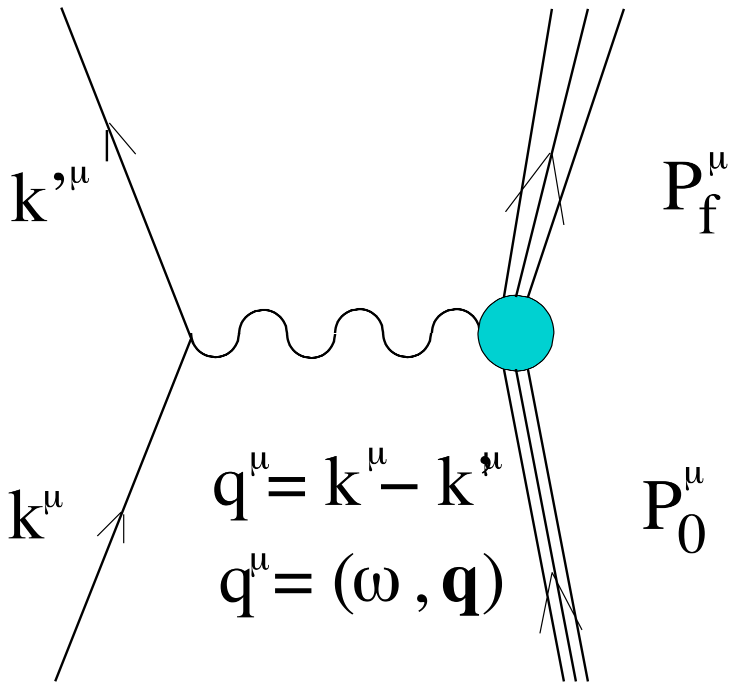
Inclusive cross section A(e,e')X

$$\frac{d^2\sigma}{d\Omega d\omega} = \sigma_M \left[\frac{Q^4}{\mathbf{q}^4} R_L(\omega, \mathbf{q}) + \left(\frac{Q^2}{2\mathbf{q}^2} + \tan^2 \frac{\theta}{2} \right) R_T(\omega, \mathbf{q}) \right]$$

with $Q^2 = -q_\mu^2 = \mathbf{q}^2 - \omega^2$ and θ scattering angle

and σ_M Mott cross section

Inelastic e-Scattering A(e,e')X



Virtual Photon

$(\omega, \mathbf{q})$

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Inclusive cross section A(e,e')X

$$\frac{d^2\sigma}{d\Omega d\omega} = \sigma_M \left[\frac{Q^4}{\mathbf{q}^4} R_L(\omega, \mathbf{q}) + \left(\frac{Q^2}{2\mathbf{q}^2} + \tan^2 \frac{\theta}{2} \right) R_T(\omega, \mathbf{q}) \right]$$

$$R_L(\omega, \mathbf{q}) = \sum\!\!\!\!\!\!\int_f \left| \langle \Psi_f | \rho(\mathbf{q}) | \Psi_0 \rangle \right|^2 \delta\left(E_f - E_0 - \omega + \frac{\mathbf{q}^2}{2M} \right)$$

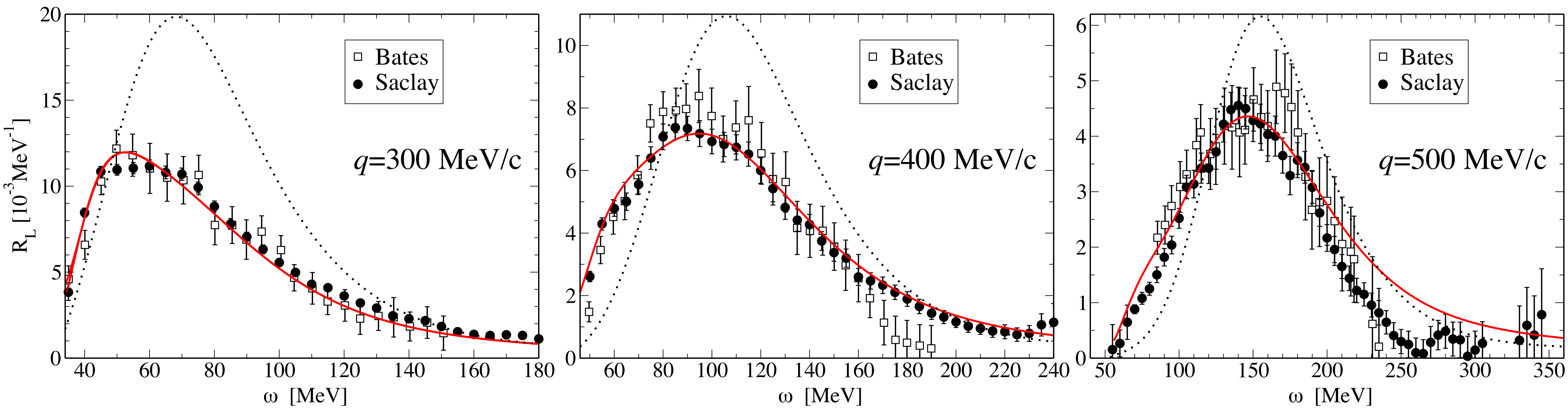
$$R_T(\omega, \mathbf{q}) = \sum\!\!\!\!\!\!\int_f \left| \langle \Psi_f | J_T(\mathbf{q}) | \Psi_0 \rangle \right|^2 \delta\left(E_f - E_0 - \omega + \frac{\mathbf{q}^2}{2M} \right)$$

Inelastic e-Scattering ^{4}He(e,e')X

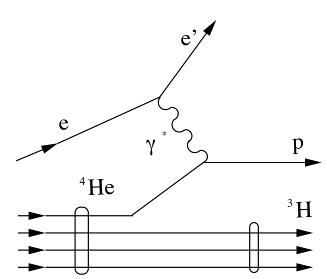
Calculation of $R_L(\omega, \mathbf{q})$ with the LIT and hyperspherical harmonics

Medium-q kinematics

S.B. et al., PRL 102, 162501 (2009)



AV18+UIX: .... PWIA



Full FSI:

- Strong effect of FSI which can be accounted rigorously with the LIT
- New project: compare ab-initio methods to account for FSI with spectral function approach (with Noemi Rocco)

Coulomb Sum Rule with CC

With Tianrui Xu, B.Sci at UBC

$$S_L(q) = \frac{1}{Z} \int_{\omega_{th}}^{\infty} d\omega \frac{R_L(\omega, q)}{{G_E^p}^2(Q^2)}$$

Total inelastic strength

$$R_L(\omega, \mathbf{q}) = \sum\!\!\!\!\!\!\int_f |\langle \Psi_f | \rho(\mathbf{q}) | \Psi_0 \rangle|^2 \, \delta\left(E_f - E_0 - \omega + \frac{\mathbf{q}^2}{2M}\right)$$

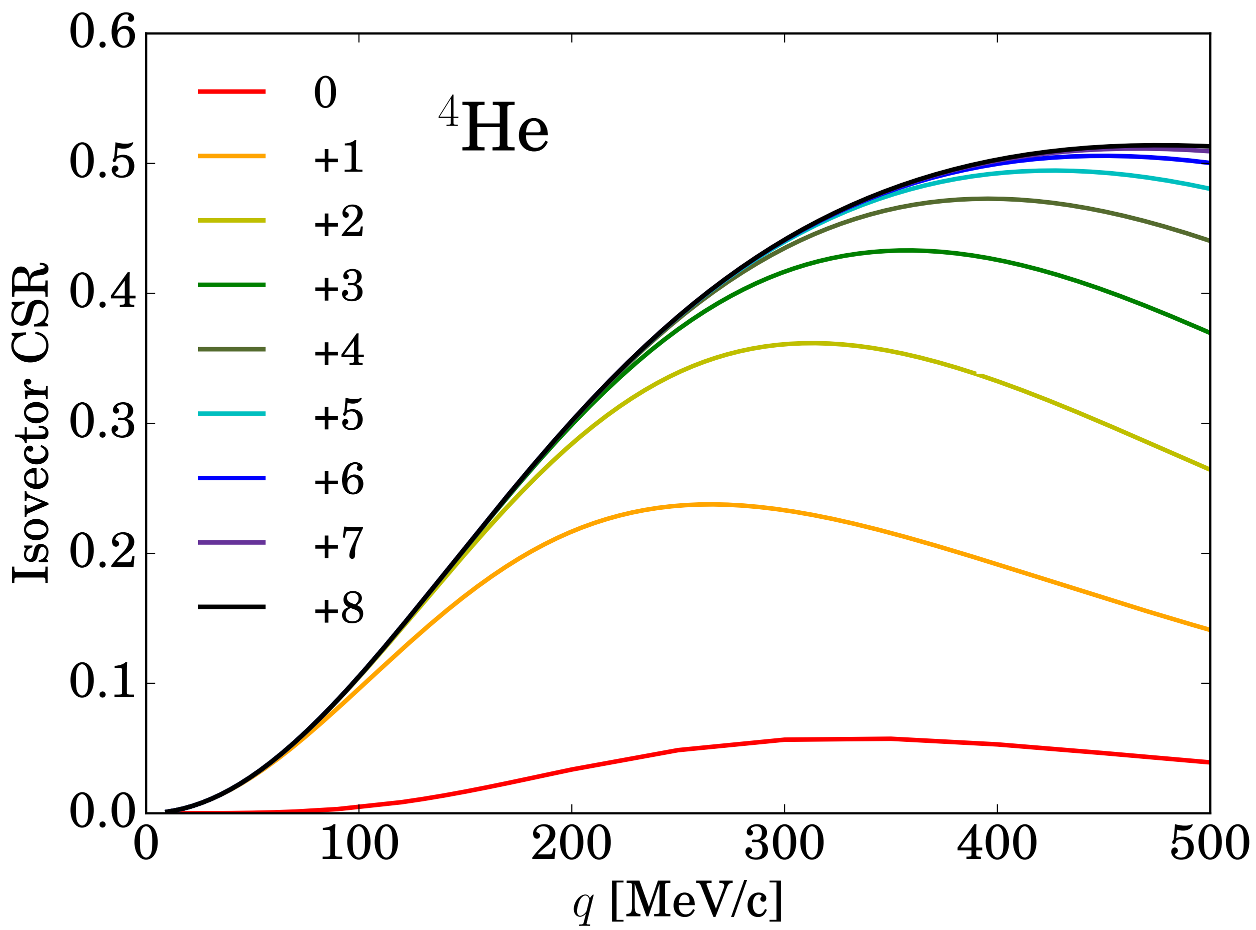
MEC are not important for $q<2\ \mathrm{fm}^{-1}$

Expand charge operator in multipoles and calculate it for different q

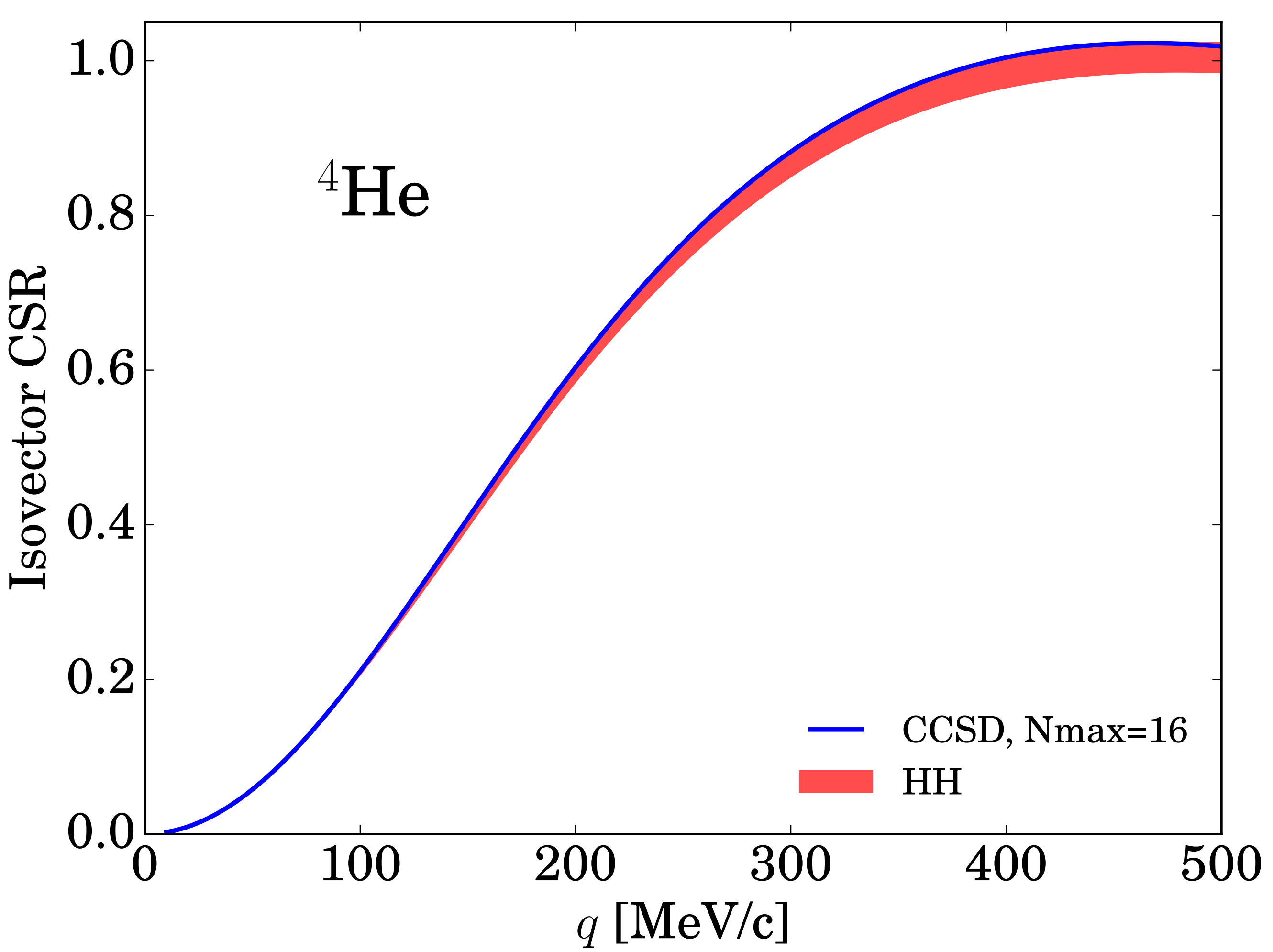
We will use a two-body force derived from chiral EFT (Entem and Machleidt N^3LO)

Coulomb Sum Rule in ^{4}He

Convergence in terms of multipoles

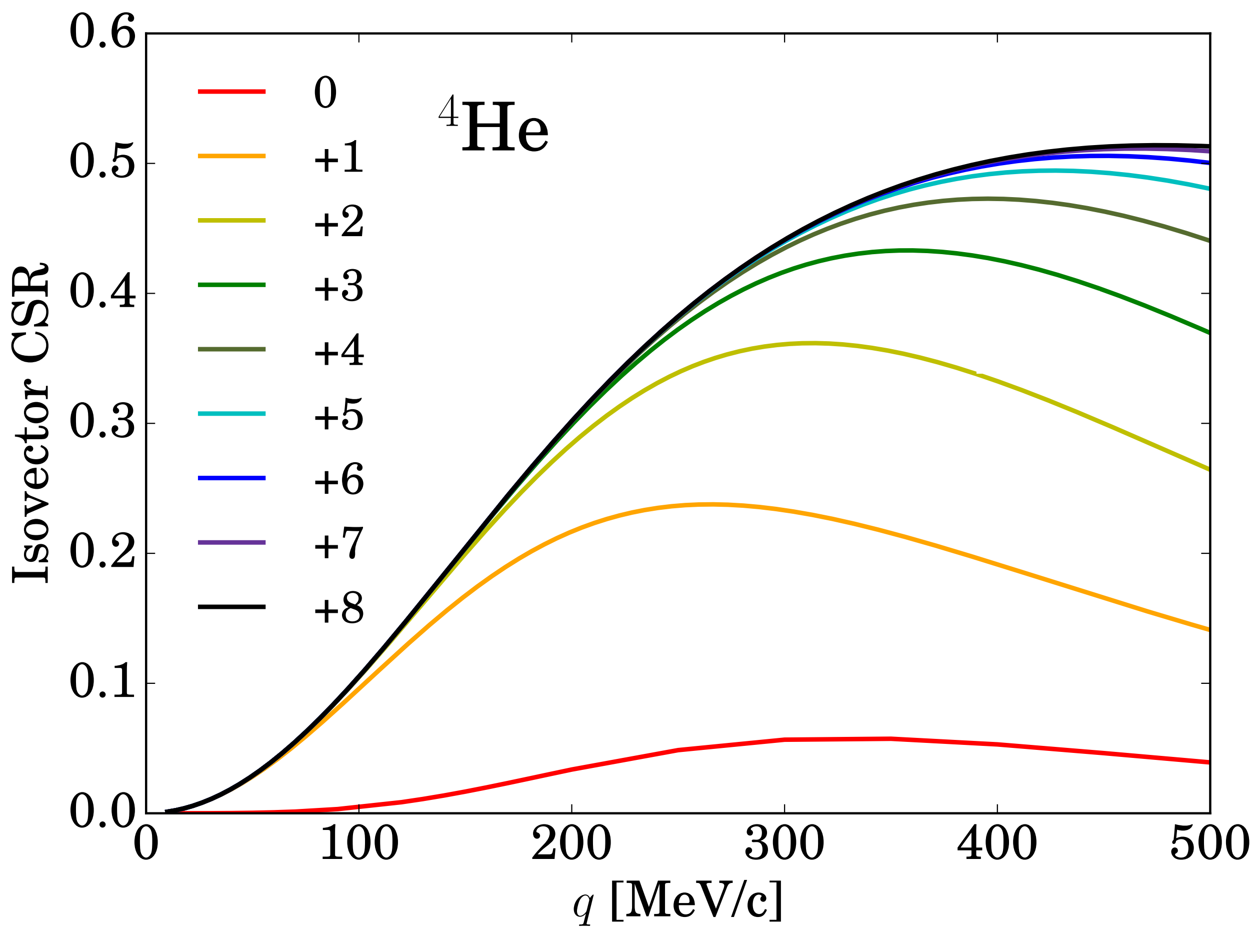


Benchmarking with HH



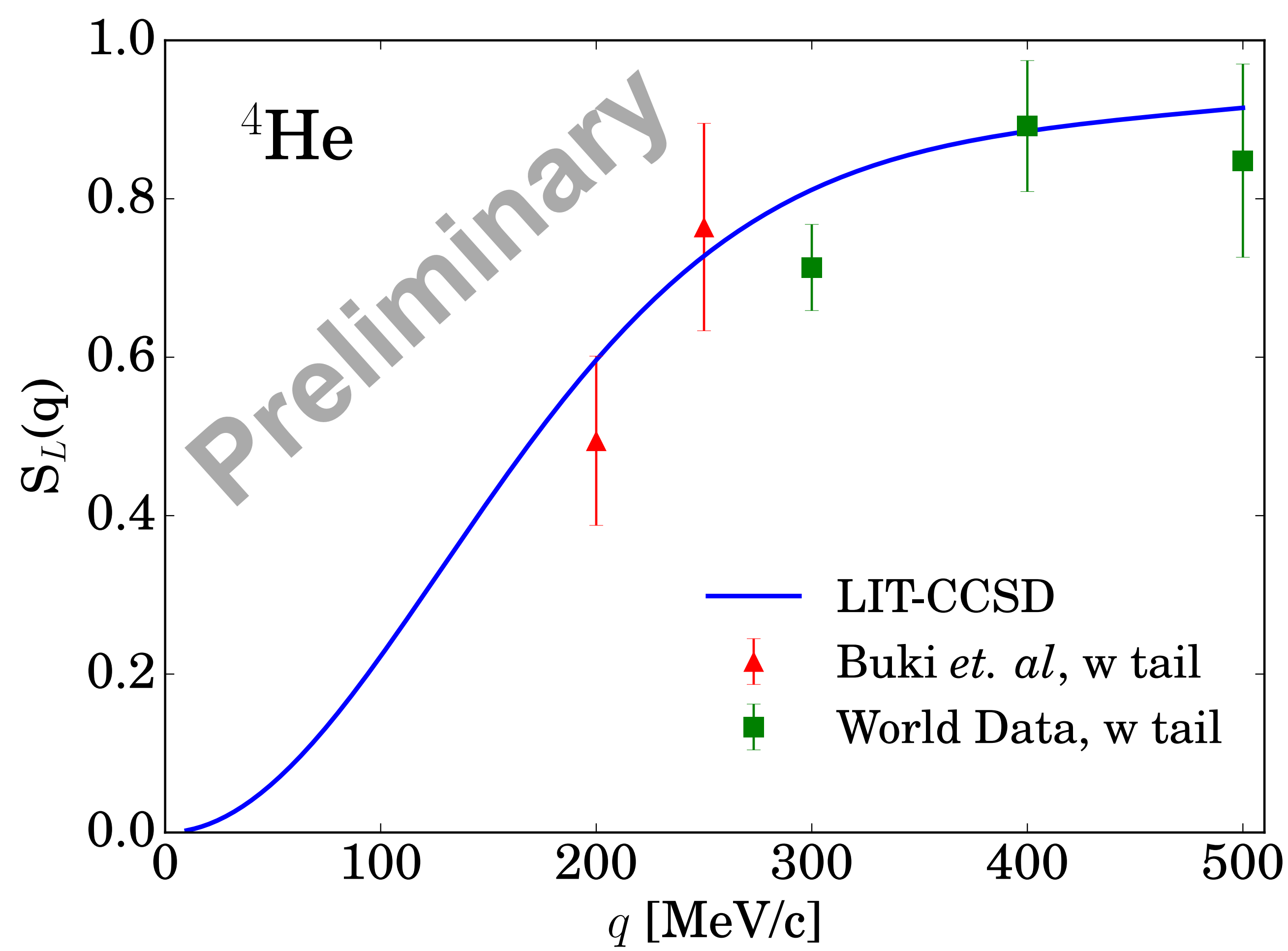
Coulomb Sum Rule in ^{4}He

Convergence in terms of multipoles



T.Xu *et al.*, EPJ Web of Conferences 113, 04016 (2016)

Comparison with experiment



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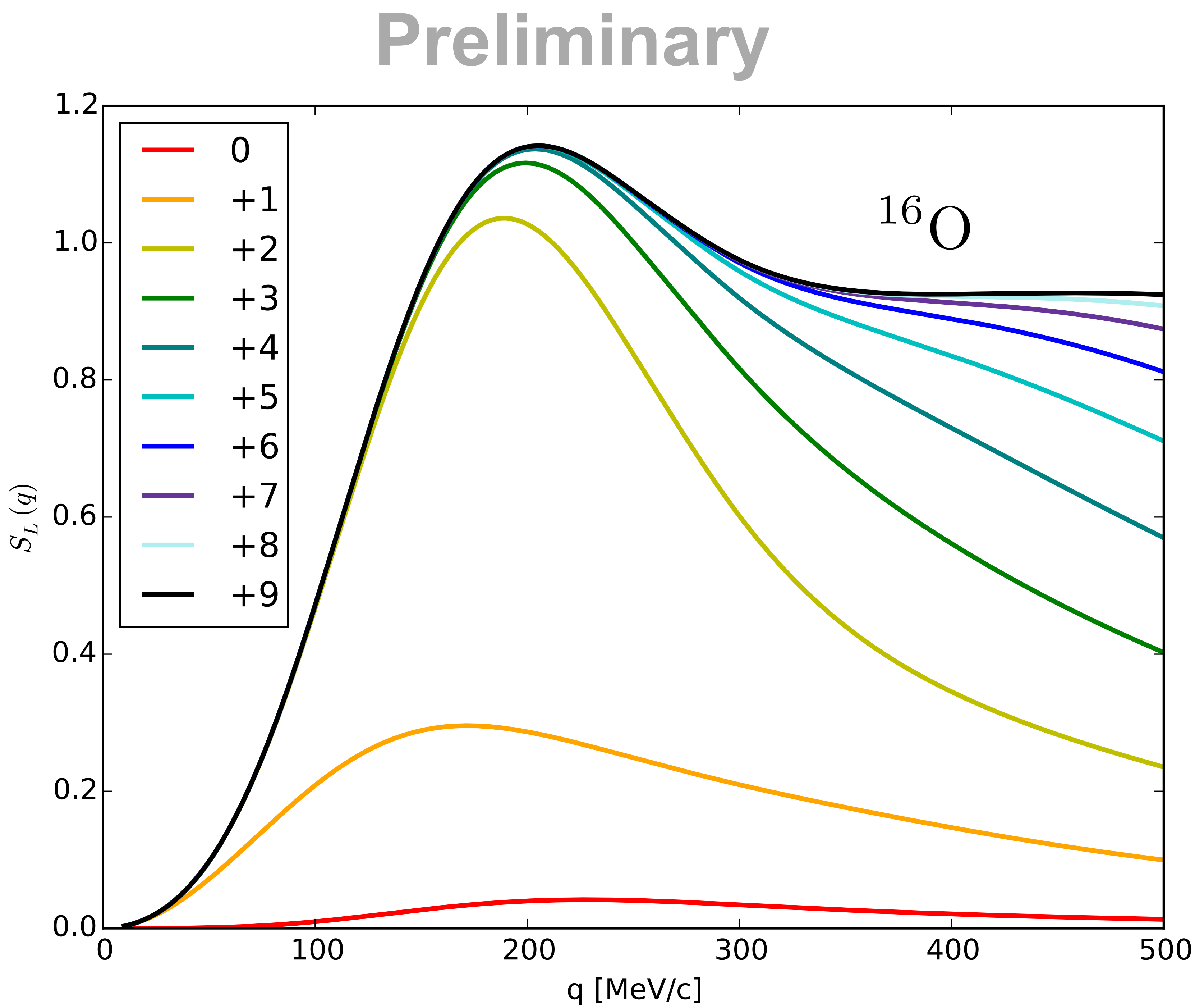
Coulomb Sum Rule in ^{16}O



Next address transverse and weak response in ^{16}O with two-body currents

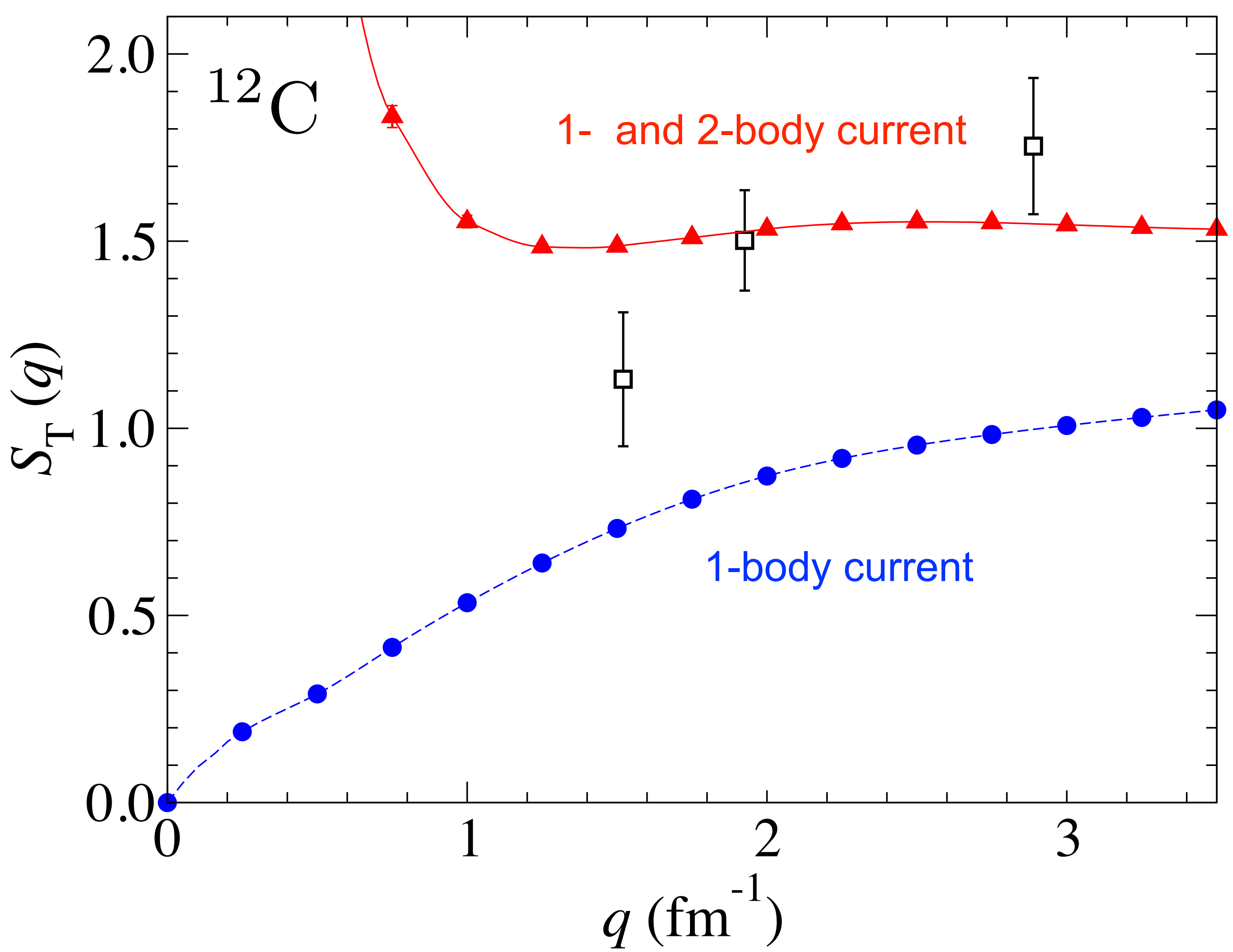
Two-body Currents

See talks by S.Pastore, A.Schwenk

Two-body currents (or MEC) play a crucial role in double beta decay and lepton-scattering

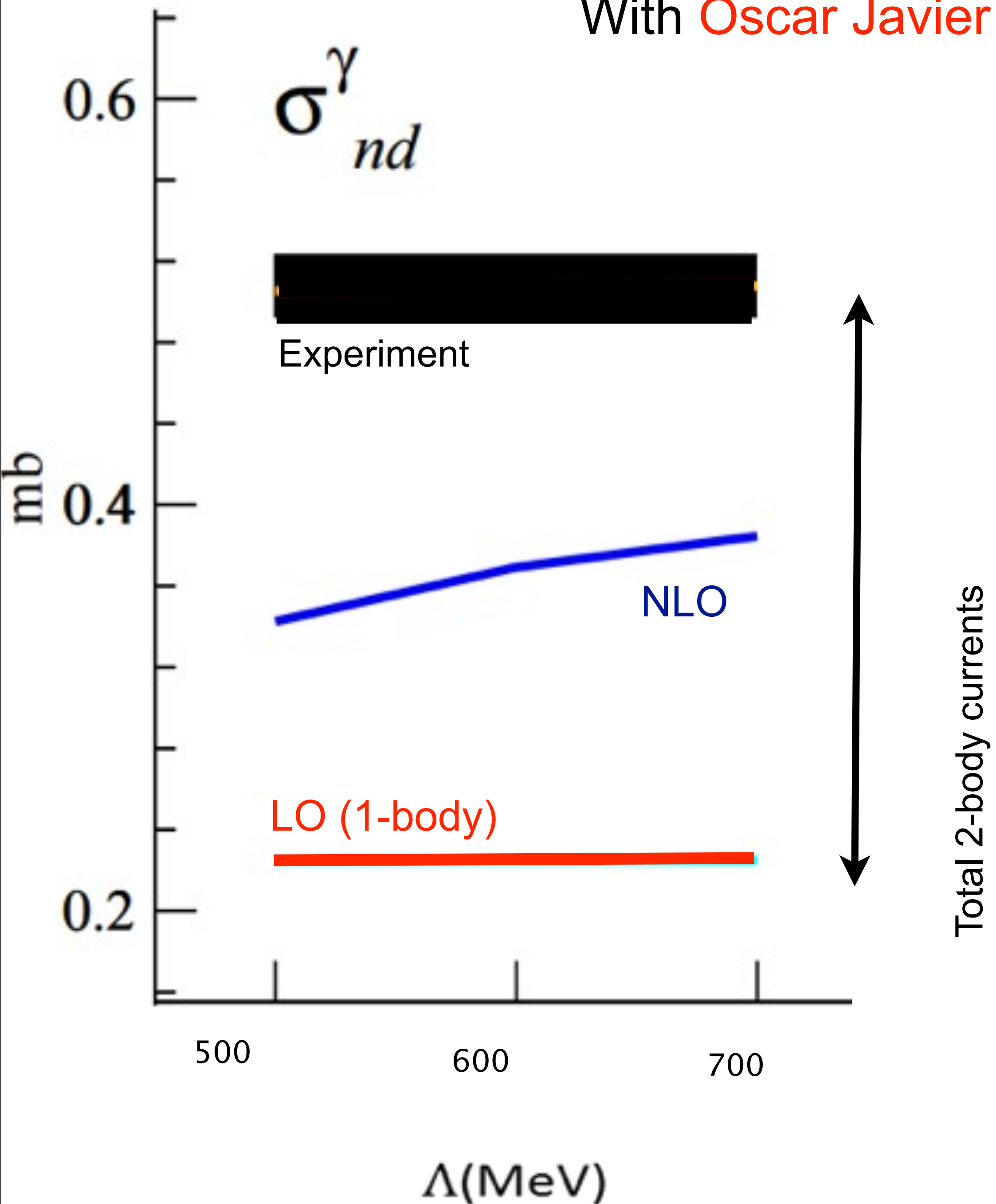
Example of the Transverse sum rule

Lovato *et al.*, PRL **111**, 092501 (2013)



Two-body Currents

With Oscar Javier Hernandez, PhD student at UBC

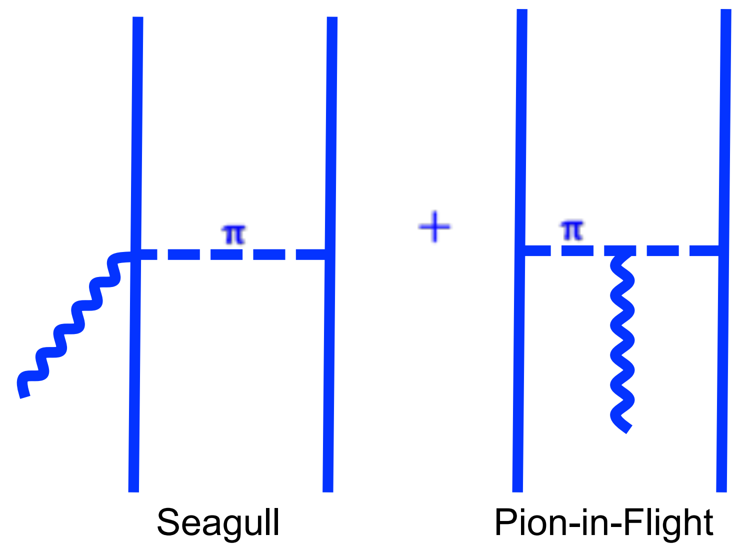


Adapted from L. Girlanda, *et al.*
EPJ Web of Conferences 3, 01004 (2010)

At NLO you get about 70% of the total effect of 2-body currents

NLO current

one pion exchange diagrams



Two-body Currents

With Oscar Javier Hernandez, PhD student at UBC

- Performed the multipole expansion of the NLO current at finite q → need for neutrino scattering

- Check the low-momentum case of magnetic dipole for the deuterium susceptibility

$$\chi_M = \frac{2\alpha}{3} \int d\omega \frac{R_M(\omega)}{\omega} \qquad R_M(\omega) = \frac{1}{2J_0+1} \sum_N |\langle N|\mu|N_0\rangle|^2 \delta(E_N - E_0 - \omega)$$

Deuteron	Potential	Current	Susceptibility [fm^3]
Our work	AV18	LO	0.0678
Friar*	AV18	LO	0.0678
Our work	AV18	NLO	0.0756
Our work	AV18	NLO+Δ	0.0776
Friar*	AV18	NLO+Δ	0.0774

*J. Friar and G.L. Payne, Phys. Rev. C 56 619 (1997)

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Deuteron	Potential	Current	Susceptibility [fm^3]
Our work	AV18	LO	$\boxed{0.0678}$
Friar*	AV18	LO	$\boxed{0.0678}$
Our work	AV18	NLO	0.0756
Our work	AV18	NLO+Δ	$\boxed{0.0776}$
Friar*	AV18	NLO+Δ	$\boxed{0.0774}$

11% enhancement

14% enhancement

*J. Friar and G.L. Payne, Phys. Rev. C 56 619 (1997)

- To do: apply to heavier nuclei using coupled-cluster theory

Magnetic Sum Rule

$$S_M = \int d\omega \; R_M(\omega)$$

Validation for ^{4}He

Comparison of coupled cluster with exact hyperspherical harmonics (EIHH) with NN forces at N^{3}LO

1-body operator Preliminary

Method	in μ_N^2
EIHH	1.69
CCSD	1.62
CCSD+(T)	1.74

Outlook

LIT + w.f. from CC + J^μ

- Possibly some of the technology developed for this program can be useful to double-beta decay program:
  - ➡ Gamow-Teller strength functions
  - ➡ Role of subnuclear degrees of freedom in quenching factors

National Research Council Canada

CINP ICPN Canadian Institute of Nuclear Physics Institut canadien de physique nucléaire

NSERC CRSNG

Thank you for your attention!